Measuring the Earth

Key Learning Content

This film tells the story of how Al-Biruni, an 11th century Persian scholar, calculated the circumference of the Earth by using a mountain and trigonometry. First, the height of the mountain was calculated by measuring its angle of elevation from two different points. Then, by measuring the angle from the top of the mountain to the horizon, the angles were calculated in a large right-angled triangle that stretched from the centre of the Earth through the mountain to the horizon. By combining these measurements it was possible to calculate the radius of the Earth, and hence its circumference, to an accuracy of more than 99%.

Familiarity with trigonometry would be useful prior to watching the film, although an understanding of the precise details of the calculations performed is not essential.

Core Outcomes

Learning Points

- Be able to understand the term 'right-angled triangle' and the angle properties of these triangles.
- Be able to understand and use sine, cosine and tangent of acute angles to determine the lengths and angles of a right-angled triangle.
- Be able to understand and use angles of elevation and depression.
- Be able to recognise the terms 'centre', 'radius', 'diameter', 'circumference', and 'tangent' of a circle.
- Be able to find circumferences and areas of circles using relevant formulae.

Suggested Activities

- Solve trigonometry problems involving angles of elevation and depression.
- Calculate the height of an object by taking its elevation from two different points.
- Solve problems involving the circumference and area of a circle.

Extension Outcomes

Learning Points

- Be able to provide reasons, using standard geometrical statements, to support numerical values for angles obtained in any geometrical context involving lines, polygons and circles.
- Be able to understand that a tangent to a circle is perpendicular to the radius at the point of contact.
- Be able to apply trigonometric methods to solve problems in three dimensions.

Suggested Activities

- Write trigonometric equations from the diagrams given in the film and solve these using algebra.
- Carry out a sensitivity analysis for the estimate of the Earth's radius relative to the accuracy of the measurements described in the film.
- Solve problems involving latitude and longitude on the Earth's surface.

Using modern scientific apparatus we can now accurately measure the dimensions of our planet.

Related Films

To use before the lesson plan:

Distance to the Sun and Moon	This film provides an introduction to trigonometry and its use in measuring relative distances.
What Do Sine Waves Sound Like?	This film explores the link between trigonometry and music through the amplitude and frequency of sound waves.

To use after the lesson plan:

Jai Singh	This film shows how Indian astronomers measured angles to an astonishing level of accuracy using basic measurement tools.
Calculating Pi: Archimedes	This film describes how ancient mathematicians estimated the value of Pi before computers and pocket calculators.
How Long is a Metre?	This film explains how the metre was once defined as a fraction of the circumference of the Earth.
Hyperbolic Geometry	This film describes how hyperbolic geometry provides an alternative to Euclidian and Elliptical Geometry, and shows that it is a more scientifically correct method of measuring distances in curved space.

Guide Lesson Plan

Introduction

Ask the students to estimate the circumference of the Earth. If no correct answer is given, provide a context: ask them to estimate it from airplane journeys, or from known dimensions of their country or continent. See how close they can get to the correct distance.

Show Film

Measuring the Earth

Main Activity

Foundation

Confirm that students know what is meant by ‘the Earth’s circumference’, i.e. the distance of any great circle on the surface of the Earth. Ask the students if it matters where the measurements in the film are made. Agree that the location of the mountain close to the sea gives an accurate reading of height above sea level. Next, revisit trigonometry relationships and define angles of elevation and depression. Then work through practical problems, e.g. find the height of a bell-tower or the height of a cliff. Progress to problems where the distance from the object is unknown (as in the film) and calculate heights using two angles of elevation at a given distance apart.

Main Activity cont ...

Advanced

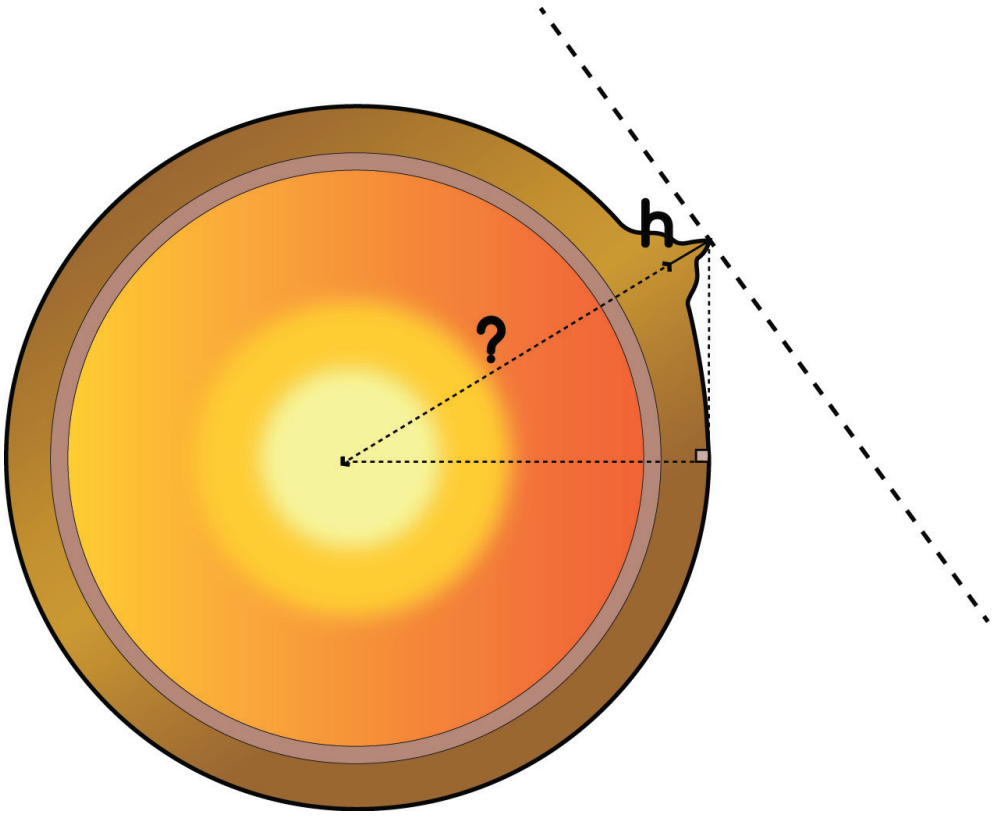
Give the height of the mountain in the film and the angle to the horizon. Ask the students to work out from this how Al-Biruni managed to draw his right-angled triangle from the centre of the Earth. What were the other angles in this triangle? Then ask them to calculate the radius of the Earth by finding the cosine of the angle at the centre of the Earth and solving the resulting equation. Finally, ask the students to calculate the effect on their answer if first, the angle to the horizon and second, the height of the mountain are out by 10%.

Extension Activity

Explain how latitude and longitude are calculated. Then set problems working out the distance between points on the Earth's surface on the same latitude but with different longitude. Calculate the radius of different circles of latitude.

Optional Extra

All the mathematics presented in the film assumes that the Earth is in the shape of a sphere. Research whether this assumption is correct and, if not, explore how this affects Al-Biruni's estimate.



The Earth's circumference can be calculated using the formula $\pi \times d$.