A Pattern in the Primes

Key Learning Content

This film tells the story of Riemann's famous but highly complex hypothesis, which relates to the number and spacing of prime numbers. Prime numbers are defined and examples given. Euclid's result (that there is an infinite number of primes) is stated, followed by Gauss' insight (that there appears to be a pattern in the way that prime numbers are spaced). The tantalising links between Riemann's hypothesis and the behaviour of atoms are mentioned, together with the possible implications of a proof for mathematics as a whole. Familiarity with the concepts of prime number and mathematical proof would be helpful to the viewer but is not essential. An understanding of the full complexities of the Riemann hypothesis is not necessary.

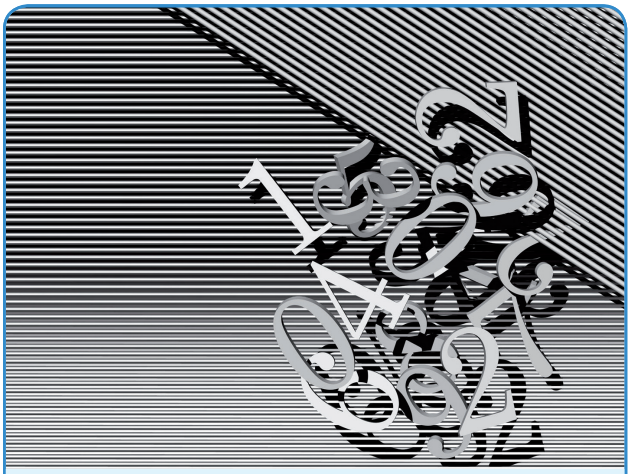
Core Outcomes

Learning Points

- Be able to understand and use the term 'prime number'.
- Be able to understand the language of probability: random, likelihood, chance and event.

Suggested Activities

- Recreate the graph shown in the film of the number of primes (y) less than a given number (x).
- Calculate the number of primes between 1 and 100; 100 and 1000; 1000 and 10,000 and so on, and confirm the probabilities shown in the film.



Prime numbers are whole numbers, which are divisible only by themselves and one.

Extension Outcomes

Learning Points

- Be able to understand the notion of mathematical proof.
- Be able to prove mathematical results by using algebra, including algebraic manipulation with brackets.
- Be able to understand and use estimates or measures of probability from theoretical models.

Suggested Activities

- Learn Euclid's proof that there are infinitely many primes.
- Prove simple Number Theory results, e.g. even + even = even, odd + odd = even.
- Explore Goldbach's conjecture and other prime number results.

Related Films

To use before the lesson plan:

Primed for Survival	This film gives an introduction to prime numbers, together with a surprising application of primes in nature.

To use after the lesson plan:

The Prime Number Code	This film shows how prime numbers are used to secure every electronic money transaction.
Proofs: Million-Dollar Maths	This film describes the various mathematical problems for which a million-dollar prize is offered to mathematicians who can find the solutions.
Number Theory: Gauss	This film gives an overview of the life and work of mathematician Carl Friedrich Gauss.
Benford's Very Strange Law	This film demonstrates that strange patterns are not found in prime numbers alone, but can also occur in ordinary numbers.

Guide Lesson Plan

Introduction

Introduce or revise the definition of a prime number. List the first few primes and look at the differences between them. Ask students if they can see any pattern.

Show Film

A Pattern in the Primes

Main Activity

Foundation

Hand out a list of all prime numbers less than 1000. Ask students to construct a graph of the number of primes (y) less than a given number (x). So there is one prime less than 3, two primes less than 4 and 5, three primes less than 6 and 7, and so on. Confirm the pattern shown in the film and explore the question: Is the distribution of primes random? Hand out a list of primes showing how many are less than 100; 1000; 10,000; 100,000; 1,000,000. Get students to calculate the actual incidence of primes and compare with the numbers given in the film. Discuss a generalisation of this pattern and how it might be proved.

Main Activity cont ...

Advanced

Explain the different types of mathematical proof (i.e. by deduction, by induction, proof by contradiction). Go through Euclid's proof by contradiction that there are an infinite number of primes. Then show how to prove other mathematical results, e.g. that even + even = even. Set students the task of proving that odd + odd = even. Go through the proof with the students. Then set proofs of:

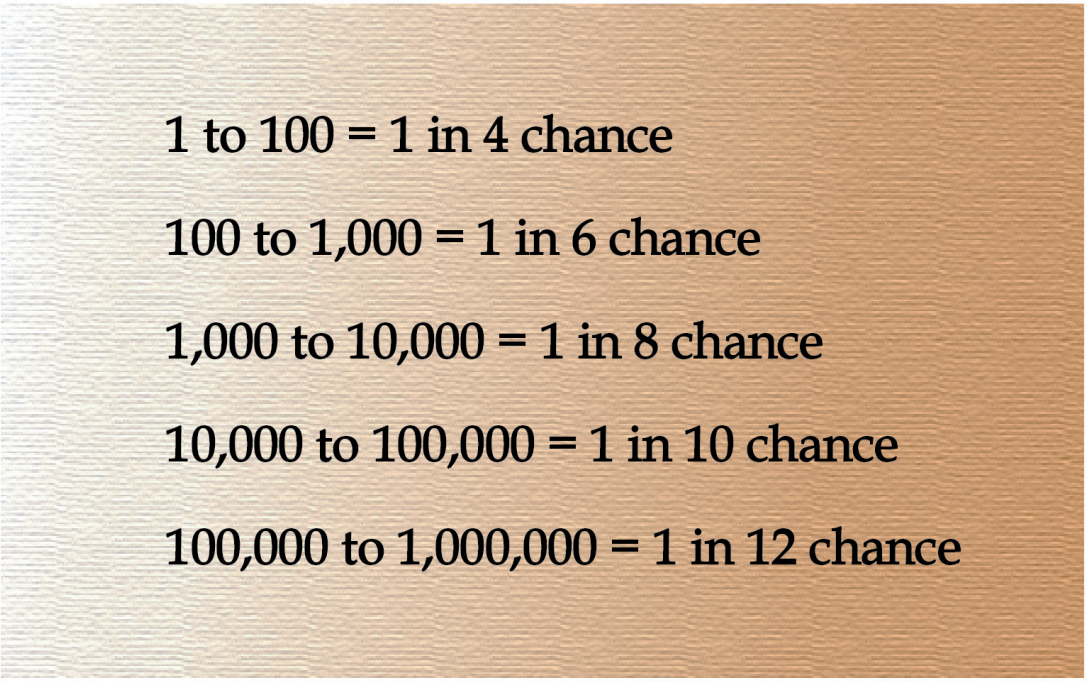
- The product of any two odd numbers is always an odd number;
- The product of any two consecutive even numbers is always a multiple of 4;
- The sum of any two consecutive odd numbers is always a multiple of 4;
- The sum of three consecutive integers is always a multiple of 3;
- The sum of two consecutive triangle numbers is a square number.

Extension Activity

State Goldbach's (unproven) conjecture, that every even integer greater than two can be expressed as the sum of two primes. Pick even numbers up to 100 and get students to demonstrate that the conjecture holds. Get students to research how far the conjecture has been checked by mathematicians using computers.

Optional Extra

Research task: Proof of Riemann's hypothesis was one of the Million Dollar Challenges. What were the others? What areas of mathematics do they relate to?



The regularity of primes decreases as numbers increase.