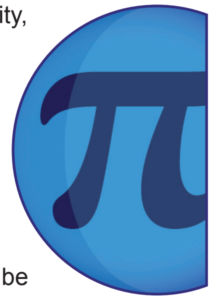
Sets: Infinity

Key Learning Content



This film explores the concept of infinity. It begins with an early Greek paradox involving infinite divisibility, before moving on to the distinction between countable and uncountable infinities. The definition of a number in terms of a one-to-one correspondence between sets is developed for finite numbers, then extended to infinite numbers. The Hebrew notation 'aleph' is introduced. Finally, an example of an infinite set that is not countable is worked through on the screen.

Familiarity with basic Set Theory is desirable prior to watching the film, in order to make best use of learning materials. Familiarity with the idea of a one-to-one function (bijection, bijective function) would be helpful, but not essential. Viewers do not need to be familiar with the distinction between countable and uncountable infinities.

Core Outcomes

Learning Points

- Be able to understand the definition of a set of numbers.
- Be able to use Set Theory to categorise types of number.
- Be able to understand the concept that a function is a mapping between elements of two sets.

Suggested Activities

- Use set language and notation to describe the different types of numbers used in mathematics, e.g. natural numbers, integers, rational numbers, irrational numbers, real numbers.
- Make statements about numbers using the set notation $\cup$, $\cap$ and $\in$. For example, 'If A is a subset of B, then write $A \subset B$.'
- Use function notation to set up one-to-one mappings between different infinite sets, e.g. from natural numbers (domain) to odd numbers (range), or from odd numbers (domain) to even numbers (range).

Extension Outcomes

Learning Points

- Be able to understand sets defined in algebraic terms.
- Be able to understand and use subsets, and the empty (or zero) set.
- Be able to understand the distinction between countable and uncountable infinities.

Suggested Activities

- Use set and function notation to show that the set of rational numbers is countable (hint: set up a one-to-one mapping between all fractions between zero and one and the natural numbers).
- Explore the number of subsets of finite sets (including the set itself and the empty set) and show that it is equal to 2 to the power n, where n is the number of elements in the set.
- Extend this result to infinite numbers (2 to the power aleph zero equals aleph one…)

Related Films

To use before the lesson plan:

Set Theory: Cantor	This film describes the life and work of George Cantor, the founder of Set Theory.
Venn Diagrams: Global Habitats	This film provides an introduction to Venn diagrams and basic set notation such as ∪ and ∩.

To use after the lesson plan:

Calculus: Newton	This film explains how Newton developed Calculus to explore rates of change, a theory that has infinity at its core.
Can Monkeys Write Shakespeare?	This film ponders the odd things that happen if you give monkeys an infinite amount of time and space.
The Biggest Number Ever	This film asks: What is the biggest finite number used as part of a formal mathematical proof?
Fractals: The Koch Snowflake	This film gives an example of the wonderful patterns made when objects exhibit infinite self-similarity.

Guide Lesson Plan

Introduction

Describe one of the ancient Greek paradoxes around infinity (e.g. Zeno's Arrow or Achilles and the Tortoise) and ask students to explain the apparent paradox. Agree that infinity is a difficult concept in mathematics.

Show Film

Sets: Infinity

Main Activity

Foundation
List the different types of number sets that students know, and describe relationships between them using set notation. Summarise using a Venn diagram, thus illustrating the relationships between the sets of natural numbers, integers, rational numbers, irrational numbers and real numbers.

Describe relationships or mappings between the natural numbers and sets or subsets of other numbers using ordinary language. Conclude that these sets or subsets are countably infinite.

Main Activity cont ...

Advanced

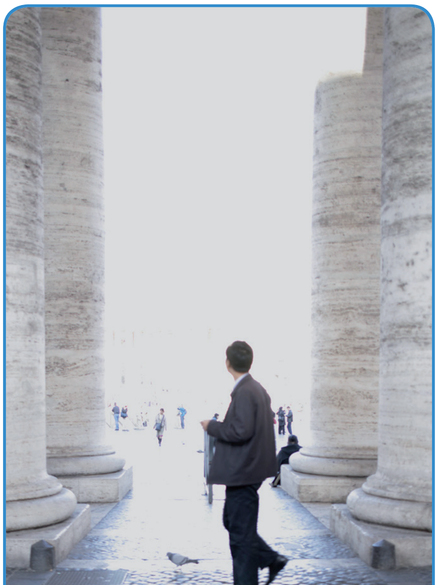
Explore the number of subsets of a set with n elements (include the set itself and the zero set). Get students to list subsets for n = 0 to 5 and check that the number of subsets = 2^n. Ask students how they would prove this result for any value of n (hint: binary).

Extension Activity

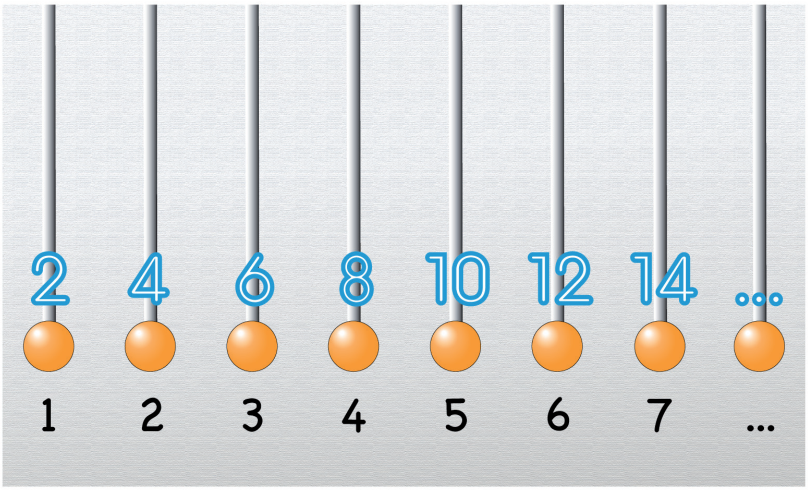
Describe function notation and the characteristics of the domain and range of a function. Define the different types of mapping between two sets and the special characteristics that make a mapping a function, and a one-to-one function. Express sets in algebraic terms and the mappings between them using function notation. Deduce what it is for a set to be countably infinite using set and function notation alone.

Optional Extra

If you have nothing, you have the zero set, Ø or { }. If you have the zero set, you can have a set with one element, the zero set: {Ø}. So you can now have a set with two elements, the zero set, and the set containing the zero set: { Ø, {Ø} }. Explore how mathematicians used this approach to build up the whole number system 'from nothing'.



The concept of never-ending sets of numbers has fascinated mathematicians through the ages, and the early Greeks were puzzled by the concept known as infinity.



There are countably infinite even numbers.