The Emperor's Chess Board

Key Learning Content

This film introduces exponentials, or powers, using the story of the Chinese emperor who wanted to reward the inventor of the game of chess. The inventor asked for a grain of rice on the first square of the chess board, two on the next, then four, then eight and so on, until the final, 64th square. The emperor agreed to the reward without realising that the total number of grains of rice was greater than all the grains of rice in China.

Exponentials are defined and the typical shape of an exponential curve is shown. The resulting geometric sequence of grains of rice is shown on the chess board, although sequences themselves are not defined.

Core Outcomes

Learning Points

- Be able to understand the terms 'exponential'
- and 'power'.
- Be able to calculate squares, cubes and higher powers.
- Be able to use index notation and index laws for multiplication of positive integer powers.
- Be able to recognise the shape of an exponential curve.

Suggested Activities

- Work out all the powers of 2, 3, 4... and so on that are less than 1000.
- By adding or subtracting powers, calculate $2^5 \times 2^3$, $2^9 \div 2^5$ and similar.
- Explain why mathematicians say that $n^0 = 1$ for
- any number n.
- Plot $y = 2^x$, $y = 3^x$, $y = 4^x$ ...on a graph and work out the pattern in the curves.

Extension Outcomes

Learning Points

- Be able to recognise geometric sequences and apply formulas for nth term and the sum of the first n terms.
- Be able to find the sum to infinity of a convergent geometric sequence.
- Be able to express integers as the product of powers of prime factors.

Suggested Activities

- Derive the formulas for the nth term and the sum of the first n terms for a geometric sequence.
- Solve simple problems involving geometric sequences.
- Recreate the chess board sequence but starting with one and halving to find the next term.
- Work out all the integers from 1 to 100 that can be written as the product of powers of 2 and 3.

The inventor of chess used his knowledge of numbers to outsmart an emperor of ancient China.

Related Films

To use before the lesson plan:

The Incredible Strength of Ants	This film reveals how number powers can explain why ants are able to lift so much relative to their body weight.

To use after the lesson plan:

What Does the Internet Weigh?	This film introduces standard form, a practical way of using numbers to express very small or very large quantities.
The Egyptians and Multiplication	This film describes an ancient method of multiplication based on powers of two.
Can Monkeys Write Shakespeare?	This film demonstrates that doing the same thing repeatedly can have surprising results.

Guide Lesson Plan

Introduction

Ask students what is the biggest number they can make with the digits 2 and 7 using basic arithmetic functions. List possible answers, from 7+2 to 7x2 to 72. See if anyone can beat $2^7 = 128$.

Show Film

The Emperor's Chess Board

Main Activity

Foundation

Get students to draw their own versions of the chess board, showing the numbers of grains of rice on each square as a power of two. Then tell them to pick any square, move seven squares on, and see what happens to the power. Then pick any square, move five squares back, and see what happens to the power. Derive the rules for multiplying and dividing powers of the same number. Set exercises to practise applying the rules.

Advanced

Ask students what the sum, S, of all the grains of rice on the chessboard would be. Agree with them that this is not an easy calculation to perform without a trick or shortcut. Then ask, what would happen to the total if you doubled all the grains of rice on each square? Suppose you then subtracted the first total from this total, term by term. Show that all chessboard squares apart from the first and last would effectively 'cancel out'.

Write this algebraically as: $2S - S = S = 2^{64} - 1$

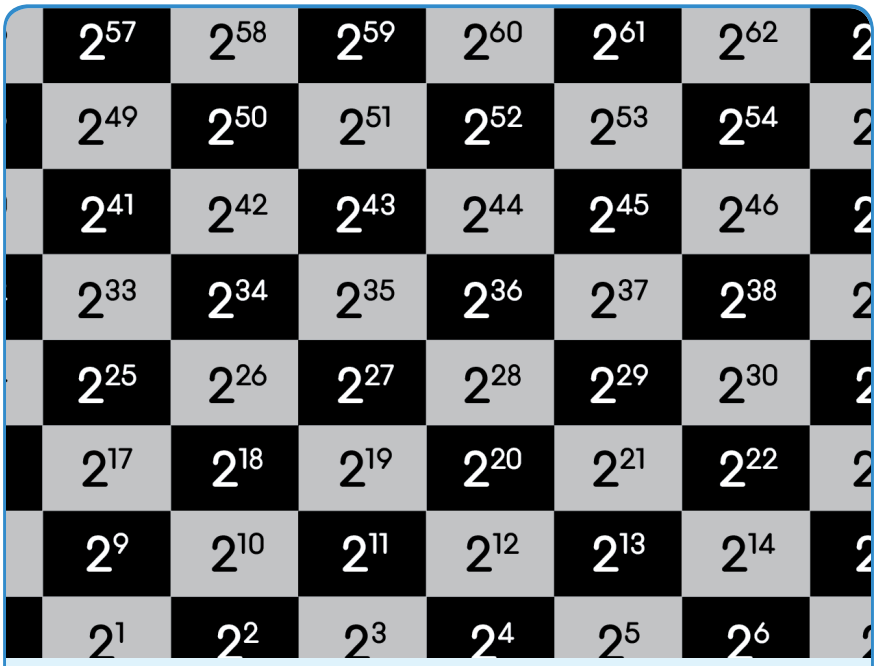
Generalise this result for any geometric sequence and set exercises to practise applying the formula.

Extension Activity

Access a spreadsheet programme. Get students to recreate the chessboard and its grains of rice using the spreadsheet, so they have an 8x8 array of numbers, increasing by x2 from square to square. Using the spreadsheet functions, get them to add up the sum of each row, and the sum of each column. Then work out the relationships between consecutive row totals, and between consecutive column totals. Justify the relationship in terms of powers of 2. Can students find any relationship between the sum of diagonals on the chess board?

Optional Extra

The rules for multiplying and dividing powers of the same number are the same as the rules for multiplying and dividing using logarithms. Research logarithms: what are they, how do they work, and what are they used for?



A pattern that increases by a constant factor each time is written using a power.