## The Fibonacci Sequence

## Key Learning Content

This film tells the story of the Fibonacci Sequence and how, according to legend, it was derived from the study of rabbits breeding. Other applications of the sequence in nature are then shown, for example, the growth of flowers and trees and the spiral of a snail's shell.

Prior familiarity with simple number sequences is helpful but not required while watching the film.
No knowledge of algebra is assumed, however sophisticated algebra can be used as extension work to explore the Golden Ratio.



- Be able to use the rules of addition and division.
- Be able to generate terms of a sequence by using term-to-term (recursive) definitions of the sequence.
- Be able to use ratio notation.


## Suggested Activities

- Generate terms of the Fibonacci Sequence.
- Explore the ratio of successive terms in the Fibonacci Sequence and show that the ratio converges to approximately 1:1.6 (the Golden Ratio).
- Identify patterns in other simple number sequences and describe them in terms of a verbal or algebraic rule.



## Extension Outcomes

## Learning Points

- Be able to manipulate algebraic fractions where the numerator and/or the denominator are numeric or linear.
- Be able to form and solve quadratic equations from data given in context.


## Suggested Activities

- Use algebra to analyse the ratio of successive terms by setting up and solving a quadratic $\left(x=1+1 / x\right.$ gives $\left.x^{2}-x-1=0\right)$. Show that the exact value of the Golden Ratio is $1:(1+\sqrt{ } 5) / 2$
- Ask the students: What is the Golden Rectangle and where is it found?
- Ask the students to demonstrate that the Golden Ratio also appears in a pentagon, and derive its exact value using geometry and algebra. What other forms of spiral are there?
- Ask the students to research Fibonacci and his other work in the field of mathematics.


## Related Films

To use before the lesson plan:

## The Beauty Formula

To use after the lesson plan:

The History of the Golden Ratio

Maths and the Mona Lisa

## A Pattern in the Primes

This film describes the rules for beauty in the human face, from symmetry to the Golden Ratio. It raises the question: Where does the Golden Ratio come from?

This film shows how the Golden Ratio has appeared in art and design throughout history.

This film analyses the Mona Lisa's proportions, in term of the Golden Ratio or Golden Rectangle.

This film describes the complicated sequence patterns that may exist between prime numbers.

## Guide Lesson Plan

## Introduction

Show a simple number sequence, for example, $4,7,10,13 \ldots$ and ask the students: What is the next term? What is the rule? Ask the students to devise their own sequences by repeatedly applying the same rule.

## Show Film

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## The Fibonacci Sequence

## Main Activity

## Foundation

Ask the students to generate the first 20 terms of the Fibonacci Sequence. Ask them to find and explain other patterns in the sequence (e.g. every third term is even).

Show other common sequences and ask them to find the next term and the rule.

## Advanced

Explain what is meant by a 'ratio'.

Ask the students to calculate the ratio between successive terms of the Fibonacci Sequence and look for a pattern (this should converge to approximately 1.6, i.e. the Golden Ratio).

Ask the students to pick any two starting numbers and generate their own sequence using the Fibonacci rule, and again calculate the ratio of successive terms - what do they notice?

## Extension Activity

Show how the Golden Rectangle is constructed and create the spiral shown in the film.

Show how to draw a regular pentagon then ask the students to work out the ratio between a side and an internal diagonal. Ask the students: Why does this relationship occur so frequently in nature?

## Optional Extra

Ask the students to research Fibonacci and his links with the Hindu number system, Arabic algebra and Renaissance mathematics.


Visualised as a sequence of squares, each the width of the previous two added together, Fibonacci's sequence is instantly recognisable.

