Tessellated Designs

Key Learning Content

This film begins by defining a tessellation and showing examples in various contexts. Regular polygons are defined and possible tessellations using regular polygons are shown. The sum of angles meeting at a point is used to demonstrate that there are only three possible tessellations of regular polygons. Examples of Esher's more exotic tessellations with irregular shapes are shown, and the link with symmetry is made. Knowledge of polygons is desirable prior to watching the film.

Core Outcomes

Learning Points

- Be able to understand the terms 'isosceles', 'equilateral' and 'right-angled' triangles and the angle properties of these triangles.
- Be able to recognise and give the names of polygons, to include parallelogram, rectangle, square, rhombus, trapezium, kite, pentagon, hexagon and octagon.
- Be able to understand that two or more polygons with the same shape and size are said to be congruent to each other.
- Be able to recognise line and rotational symmetry.

Suggested Activities

- Construct tessellations with multiple regular polygons and with one or more irregular polygons.
- Construct tessellations with drawings of recognisable objects.
- Categorise tessellations by their rotational and line symmetry.

Extension Outcomes

Learning Points

- Be able to understand the term regular polygon and calculate interior and exterior angles of regular polygons.
- Be able to understand and use the angle sum of polygons, e.g. For a polygon with n sides, the sum of the interior angles is $(2n - 4)$ right angles.

Suggested Activities

- Prove that only three regular polygons can tessellate by calculating the interior angles of polygons.
- Explore which irregular polygons will tessellate, either on their own or with other polygons.

A tiling that uses one or more similar shapes is called tessellation.

Related Films

To use before the lesson plan:

The Mirror Lines of the Taj Mahal	This film describes how the desire for pattern and symmetry produced the beautiful Taj Mahal.

To use after the lesson plan:

Bees and Their Hives	This film explains how bees choose the optimum tessellation for their hives based on storage of honey and use of wax.
Strengthening the Bank of China	This film shows how triangular tessellations are still found in architecture today, for practical as well as aesthetic reasons.
Escher and the Endless Staircase	This film looks at patterns from Escher and others which deceive the eye and undermine our assumptions about how things appear.

Guide Lesson Plan

Introduction

Ask students whether they think animals can count. Next, ask whether animals can do geometry. (Assume the answer will be 'no'.) Then show a picture of a beehive and ask students how they think bees are able to do this. Get students to draw a hexagonal tessellation by hand and compare it to the bees' efforts.

Show Film

Tessellated Designs

Main Activity

Foundation

Explain that although there are only three tessellations of single regular polygons, there are many more tessellations involving multiple shapes, both regular and irregular. Get students to construct as many tessellations as they can with regular and irregular polygons (e.g. trapezia, isosceles triangles, octagons and squares, triangles and squares and hexagons). Then show how to construct irregular tessellations by taking a square, cutting a shape out of one side and pasting it to the opposite side, joining edge to edge; then repeat for the other pair of sides. (The shape so formed will always tessellate; the trick is to get it to look like a recognizable shape.)

Advanced

Get students to calculate the interior and exterior angles of regular polygons of up to 20 sides. Form an equation in terms of n, the number of sides, for these angles. As n tends to infinity what happens to the angle and what happens to the shape? Revise the proof that only three regular polygons tessellate, using this table.

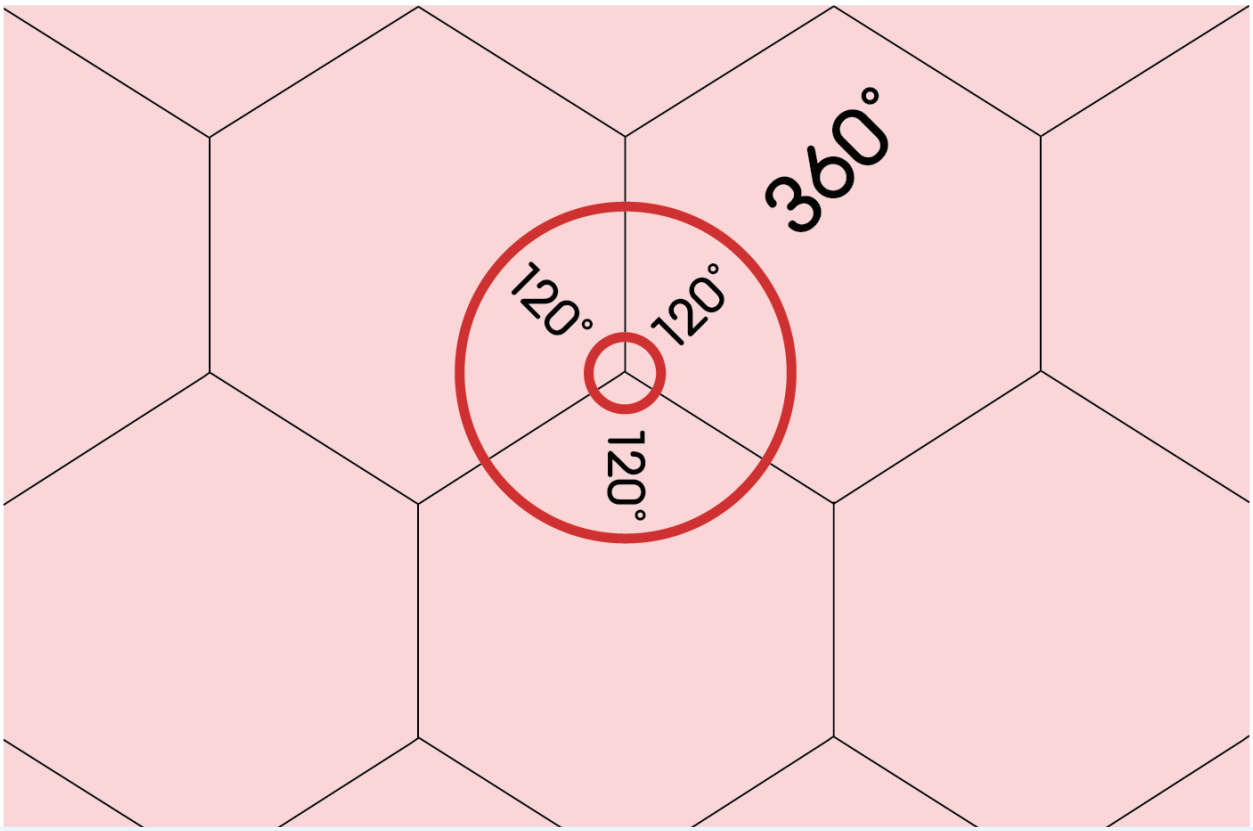
Research, using the internet, which convex irregular polygons tessellate on their own. (There are surprisingly few, although the precise number is still a matter of debate.)

Extension Activity

Show examples of Escher's complex irregular tessellations and analyse them in terms of rotational and reflective symmetry. Try to find regular polygon patterns behind the irregular tessellations.

Optional Extra

A 'polyomino' is defined as a polygon made of identical squares joined along their edges. Two squares make a domino, three squares a tromino, and four squares a tetromino. Confirm that there is only one shape of domino, two shapes of tromino, but five different tetrominoes. What are they, and which combinations tessellate?



Each angle can be determined by dividing 360° by the number of shapes meeting.