Fractals: The Koch Snowflake

Key Learning Content

This film describes at a high level what a fractal is, and how fractal-like shapes are found in nature. It focuses on the Koch Snowflake fractal, from its construction out of an equilateral triangle, to its bounded area but infinite perimeter. Although fractals can be a very complex area of mathematics, this film assumes no prior knowledge beyond familiarity with triangles and fractions. It can be used as an introduction to more complex topics such as geometric progressions and sums to infinity.

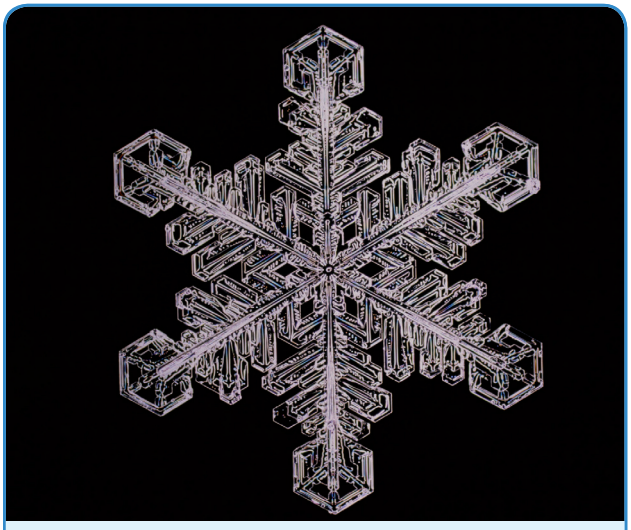
Core Outcomes

Learning Points

- Be able to understand the nature of a fractal, and its self-similarity.
- Be able to understand the geometrical properties of similar equilateral triangles.

Suggested Activities

- Work out from first principles the perimeter and area of each iteration of the Koch Snowflake.
- Create fractal patterns by repeatedly applying a simple rule to a simple shape.



The Koch Snowflake is a shape with a finite area, but an infinitely increasing perimeter.

Extension Outcomes

Learning Points

- Be able to use fraction multipliers to generate a geometric sequence.
- Be able to provide reasons, using standard geometrical statements, to support numerical values for lengths and areas in geometrical problems, and relate this to the properties of similar shapes.

Suggested Activities

- Derive algebraic expressions for the perimeter and, if possible, the area for the nth iteration of the Koch Snowflake.
- Derive the formula for the sum of the first n terms of a geometric sequence and apply this to the Snowflake perimeter.
- Explore the rotational and reflective symmetry of the Snowflake.

Related Films

To use before the lesson plan:

The Tunnel of Samos	This film shows how the ancient Greeks used basic geometry and right-angled triangles to successfully construct a tunnel under a mountain.

To use after the lesson plan:

Fractals: The Menger Sponge	This film is similar to The Koch Snowflake but explores a 3D fractal shape.
Sets: Infinity	This film provides a comprehensive introduction to the history of infinity paradoxes, and an overview of how modern mathematicians have sought to treat infinity as just another number.
Chaos By Mistake	This film explains that fractal patterns can sometimes appear chaotic, and small changes in the way they are set up can impact significantly on the final shape.

Guide Lesson Plan

Introduction

Fractals are a very visual part of mathematics and perhaps the best way to introduce them is to show examples – simply typing 'fractal' into an image search engine on the web will provide dozens of stunning images. After exploring these, ask the question: How are fractals formed?

Show Film

Fractals: The Koch Snowflake

Main Activity

Foundation

Revise how the Koch Snowflake was formed, then get students to generate their own fractal patterns. Start with simple shapes (a line, a triangle, a square) and repeatedly apply simple rules, as in the Koch example. For interesting examples, get students to try to work out expressions for length or area of their fractals.

Advanced

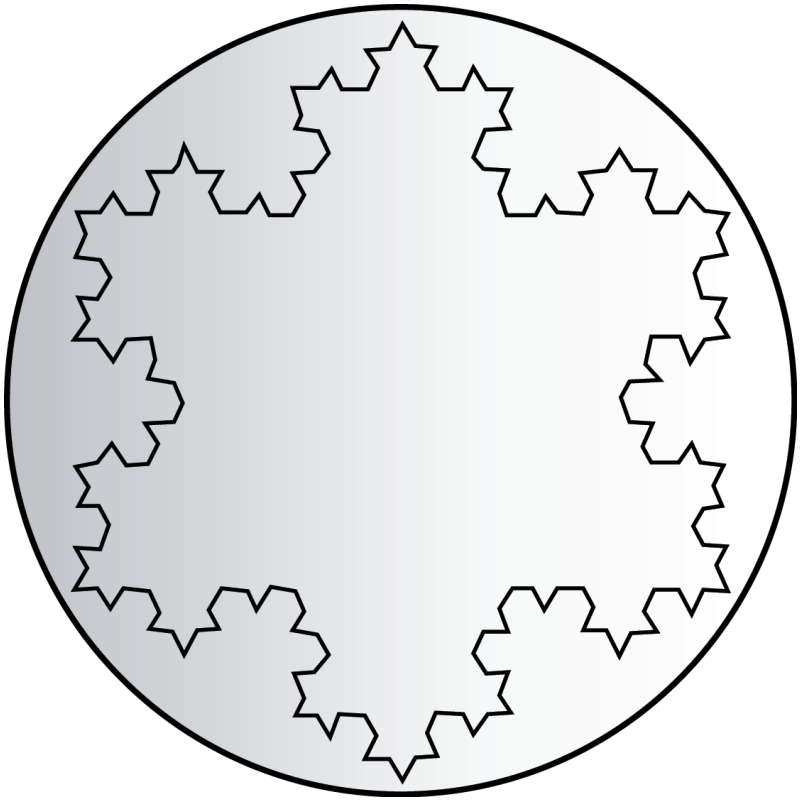
Starting with an equilateral triangle of side 1, get students to tabulate and then graph the perimeter and area of the snowflake after each iteration. Then by expressing area and perimeter algebraically, in terms of n iterations, generate formulas for perimeter and area in terms of n. Explore what happens when n tends to infinity.

Extension Activity

The infinite perimeter of the Koch Snowflake links to the fascinating work of the mathematician Benoit Mandelbrot on the Coastline paradox: that the length of the coastline of any island depends on the scale of the measurement. There are many web-based expositions of Mandelbrot's argument, with its counter-intuitive and provocative conclusion, that students can work through and critique.

Optional Extra

Type 'fractal applet' into a search engine on the web and explore the fractals that these applets generate. Question: What is the difference between a Mandelbrot and Julia set?



The Koch snowflake is created by continually repeating a certain rule:
1) Take an equilateral triangle.
2) Lose the middle third of each side.
3) Insert two sides of an equilateral triangle into each gap.
4) Repeat this rule for all of the new sides.

As the snowflake's perimeter grows, it remains contained inside a circle drawn around the original triangle, and thus the area is bounded.