How Origami Changed the World

Key Learning Content

This film describes the surprising link between the paper-folding art of origami and two problems concerning the ancient Greeks; trisecting an angle and doubling the volume of a cube. How do you divide any angle into three equal parts without measuring the angle? And how do you double the volume of a cube?

Euclid's axiomatic approach to geometry is briefly described, and his ruler-and-compasses method for bisecting an angle is shown on screen.

The groundbreaking work of the Italian mathematician Margherita Beloch is then described, and her paper-folding solutions to the problems are illustrated. Some applications of this approach within modern engineering are briefly illustrated.

Although the concepts and proofs are complex, the film does not require students to understand all the details of the mathematics, and can be shown as an extension to any origami-based activity.

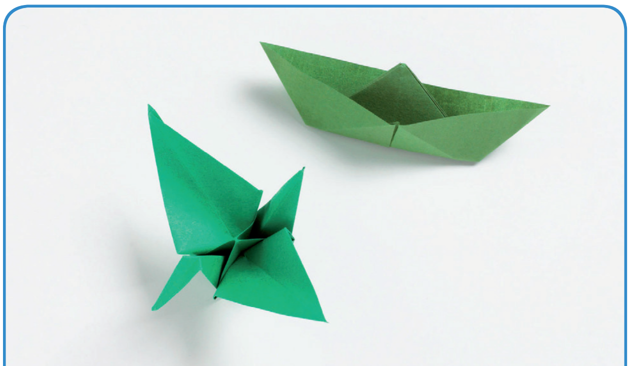
Core Outcomes

Learning Points

- Be able to use a ruler and compasses to construct the bisector of an angle.
- Be able to construct simple geometrical proofs.
- Be able to provide informal reasons when arriving at numerical solutions to geometrical problems.

Suggested Activities

- From first principles, construct the perpendicular bisector of a line segment, and the bisector of a given angle.
- Show that the perpendicular bisector of any chord drawn in a circle goes through the centre.



The revelation that origami could be used to calculate angles and volume has led to its application in modern engineering.

Extension Outcomes

Learning Points

- Be able to understand the nature of mathematical proofs, Euclid's Axioms and Pythagoras' Theory.
- Be able to understand the meaning of and manipulate surds in problem solving.

Suggested Activities

- Prove Pythagoras' Theorem using the properties of similar triangles.
- Prove the circle theorems using the properties of isosceles triangles.
- Describe Beloch's paper-folding method to another person and why it works in the two cases shown.
- Prove that the $^{2}\sqrt{2}$ is irrational.

Related Films

To use before the lesson plan:

Fractions: Pythagorean Tuning — This film provides context of ancient Greek mathematics, their fascination with finding patterns in nature, and their interest in dividing lengths and area.

Proving Pythagoras — This film shows the different ways in which Pythagoras, and others, proved this famous theorem.

Irrational Numbers: Pythagoras — This film demonstrates how the 'discovery' of irrational numbers/surds caused huge problems for the ancient Greeks (and their discomfort with the use of the $\sqrt[3]{2}$ when doubling the volume of a cube).

To use after the lesson plan:

The Incredible Strength of Ants — This film explores the strange implications of length, area and volume scale factors; linked to the problem of doubling the volume of a cube.

Proofs: Million-Dollar Maths — This film provides examples of difficult mathematical proofs that have eluded mathematicians to this day.

Hyperbolic Geometry — This film explains what happens when you don't take Euclid's Axioms as your starting point in geometry.

Guide Lesson Plan

Introduction

Assuming that the students have not yet been introduced to circle theorems or formal proofs, begin the lesson with a simple origami task.

Ask the students: What has this got to do with mathematics?

Show Film

How Origami Changed the World

Main Activity

Foundation

Describe how to bisect any given angle using only a ruler and compasses (as illustrated in the film). Ask the students to draw any angle and bisect it using the ruler-and-compasses method.

They should use a protractor to check the result.

Main Activity cont ...

Advanced

Show Euclid's Axioms and explain their purpose.

Ask the students to prove from first principles that there are 180° in a triangle (Hint: draw a line parallel to the base through the third vertex then consider alternate angles).

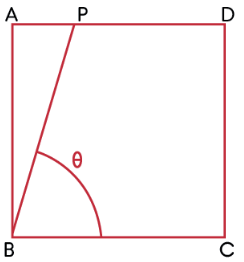
Extension Activity

Ask the students to demonstrate other geometrical results using a ruler and compasses, for example, the perpendicular bisector of a line, and the perpendicular bisector of a chord.

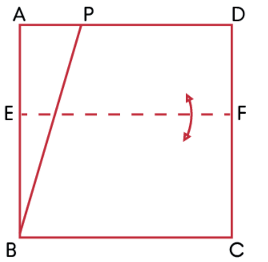
Students should then check their results using a protractor.

Optional Extra

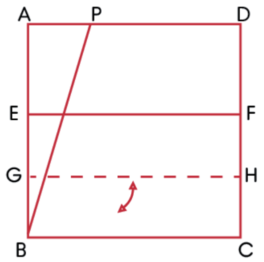
Ask the students: What angles could you easily trisect? What scale factor increases could pupils easily apply to a cube? (Trisect 270°; increase the volume of a cube by a factor of 8).



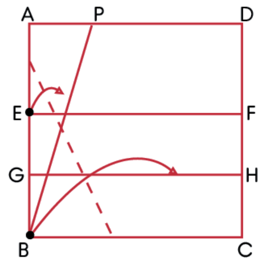
1. Draw the desired angle (PBC)



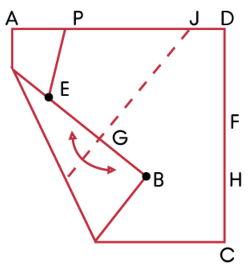
2. Make a horizontal fold anywhere across the square (EF)



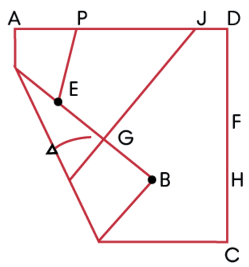
3. Fold BC up to EF and unfold to create GH



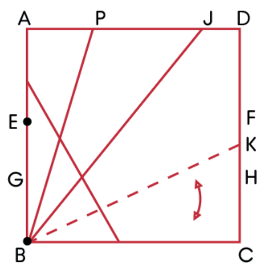
4. Fold the bottom left corner so that point E touches line BP and point B touches line GH



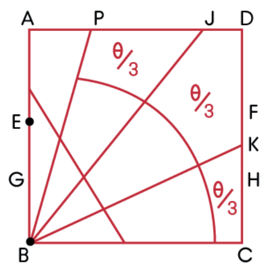
5. Without unfolding point B, continue the crease that ends at point G to create point J, then unfold



6. Unfold corner B



7. Fold line BC up to line BJ and unfold



8. The two creases BJ and BK trisect the original angle PBC

Follow the instructions to trisect an angle by folding paper.