Where is the Centre of a Triangle?

Key Learning Content

This film considers what is meant by the centre of a triangle. There are many different interpretations of, and answers to, this deceptively simple question. The film shows clearly how to construct the different centres of a triangle, and sets out some surprising properties that the centres share. Constructions involve bisecting lines and angles. The film encourages a fresh look at the properties of shapes and a rigorous approach to analysing them.

Core Outcomes

Learning Points

- Be able to understand the terms ‘isosceles’, ‘equilateral’ and ‘right-angled’ triangles and the angle properties of these triangles.
- Be able to understand the terms ‘altitude’, ‘median’, ‘perpendicular bisector’ and ‘orthogonal’.
- Be able to construct triangles and other two-dimensional shapes using a ruler, a protractor and compasses.

Suggested Activities

- Construct triangles when the lengths of three sides are given but no angles.
- Construct triangles when two sides and a contained angle are given.

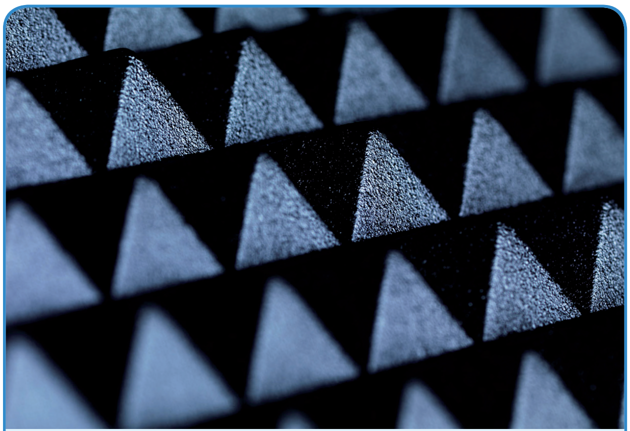
Extension Outcomes

Learning Points

- Be able to use straight edge and compasses to construct the perpendicular bisector of a line segment and the bisector of an angle.
- Be able to understand the term ‘locus’ and describe the locus of a point in simple situations.
- Be able to relate triangle properties to inscribed and circumscribed circles.

Suggested Activities

- Construct perpendicular bisectors and angle bisectors for given lines and triangles.
- Solve problems involving the locus of a point.
- Prove that the circumcentre and incentre are centres of circles touching the vertices and sides of the triangle respectively.



Within any given triangle, the orthocentre, the nine-point centre, the centroid and the circumcentre lie on one straight line.

Related Films

To use before the lesson plan:

Geometry: Euclid

This film gives the background to the Greek geometrical approach to mathematics and explores what can be achieved using only a straight edge and compasses.

Strengthening the Bank of China

This film describes how the properties of triangles make them suitable for use in construction and design contexts.

To use after the lesson plan:

How Origami Changed the World

This film shows how the problem of trisecting an angle baffled mathematicians for centuries until someone used origami to solve it.

The Seven Bridges of Konigsberg

This film describes how looking at a problem in a slightly different way helped to solve a tricky practical challenge.

Guide Lesson Plan

Introduction

Ask students to mark the centre of a circle, a square, a rectangle, and a regular pentagon. Then ask: Where is the centre of a triangle? Draw different triangles and discuss alternative answers.

Show Film

Where is the Centre of a Triangle?

Main Activity

Foundation
Explain how to draw a triangle when given three sides using only a ruler and compasses. Explain how to draw a triangle given two sides and a contained angle using a ruler and protractor. Get students to practise drawing these shapes and get them to measure the angles and sides of the triangles they construct. Construct one of the centres to each triangle.

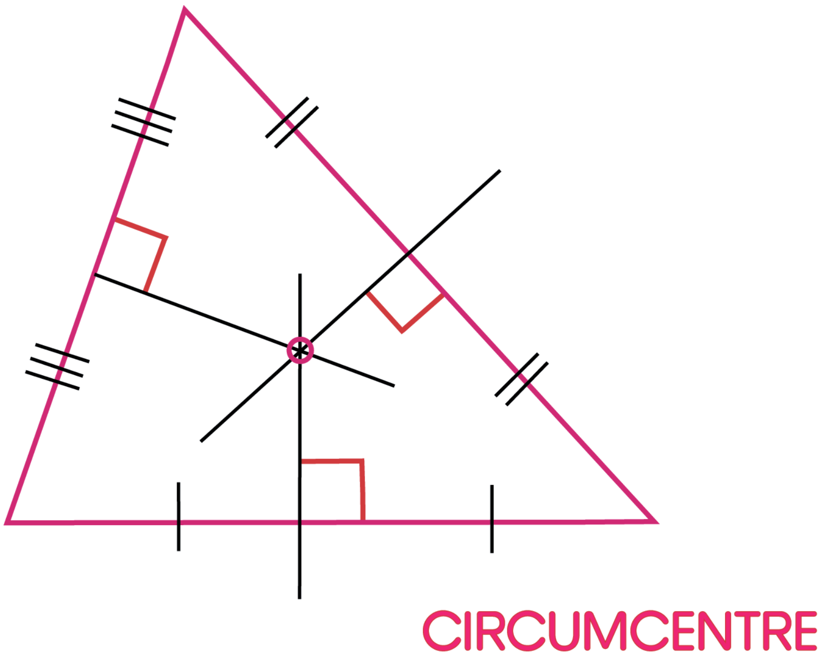
Advanced
Explain why the Greeks were concerned to find solutions to problems using only a straight edge and compasses. Demonstrate how to find perpendicular bisectors of lines and bisectors of angles using only these tools. Get students to practise these methods stressing the importance of showing clear construction lines. Use these techniques to find the incentre and circumcentre of a triangle using only a straight edge and compasses.

Extension Activity

Define what is meant by the locus of a point and set simple problems involving loci, e.g. what is the shape made by a goat eating grass in a field when tethered to a pole? Describe perpendicular bisectors and angle bisectors as loci.

Optional Extra

Construct all five centres of a triangle and confirm properties. Draw a circle through the vertices of the triangle with the circumcentre as centre, and another circle just touching all three sides with the incentre as centre. Prove that it is always possible to draw these circles whatever triangle you start with.



One of the ways in which the centre of a triangle can be found is by drawing on the three perpendicular bisectors, at right angles from the midpoint of each side. The lines cross at the circumcentre of the triangle.