Topology

Key Learning Content

This film gives a high-level introduction to topology, the branch of geometry that studies shapes that can be stretched, bent or squeezed into each other. Thus, a rugby ball is topologically equivalent to a football, and a bagel to a coffee mug. The special properties of the Mobius band are described, and examples given of topological subway maps. Although the film can be watched without any specific prior mathematical knowledge, an understanding of Euclidean Geometry and 3D shapes is useful in order to appreciate the broader mathematical context.

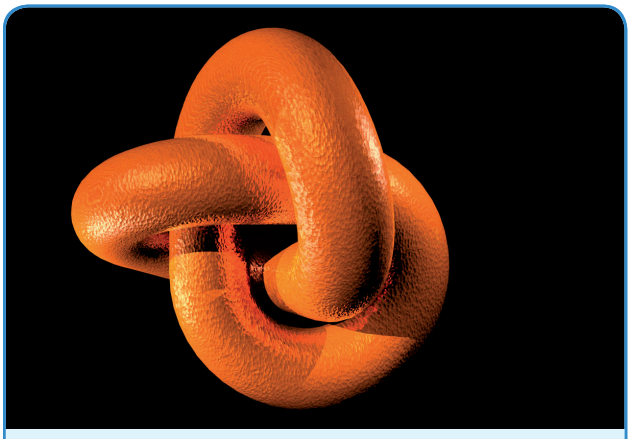
Core Outcomes

Learning Points

- Be able to understand what is meant by topological equivalence and homeomorphism.
- Be able to categorise shapes by their topological equivalence.
- Be able to interpret topological maps, and to draw a topological map from a conventional one.

Suggested Activities

- Group everyday objects by topological equivalence.
- Draw topological maps of familiar places.



Topology explores the way one object can be transformed into another by bending, stretching and squeezing, but not by cutting or tearing.

Extension Outcomes

Learning Points

- Be able to understand that rotations, reflections and translations preserve length and angle, so that a shape changed under any of these transformations remains congruent to the original shape.
- Be able to understand that stretching, bending and squeezing distort length and angle but preserve path continuity.
- Be able to identify and give complete descriptions of transformations from one shape to another.

Suggested Activities

- Categorisation of Platonic solids according to their number of faces, vertices and edges.
- Demonstration of the duality of Platonic solids.

Related Films

To use before the lesson plan:

Geometry: Euclid	This film describes the essential elements of Euclidean Geometry that underpin our intuitive understanding of space and form the basis of most geometry teaching in schools today.
Polyhedra: Platonic Solids	This film gives an introduction to Platonic solids and their properties.
Transformations: Skateboarding	This film discusses standard length transformations and angle-preserving transformations that are redundant within topology.

To use after the lesson plan:

The Seven Bridges of Konigsberg	This film describes a very famous problem solved using topology.
Networks: Labyrinths and Mazes	This film features examples of continuous paths within complex networks.

Guide Lesson Plan

Introduction

Ask the students: How many sides does a cube have? Next: How many edges are there on a tetrahedron? Then: Can you think of a shape that has only one side and one edge? The answer is a Mobius band.

Show Film

Topology

Main Activity

Foundation

Start by getting the students to make Mobius bands thus: take a strip of paper, twisting one end and sticking the ends together. Explore the properties of the shape. Get students to cut the band down the middle – do they get what they expected? Agree that the properties of shapes are sometimes far from obvious. Then talk through the equivalences of a rugby ball and football, and a bagel and coffee mug, taking care to emphasise which properties are invariant and which may change under topological equivalence. Then hand out a list of letters of the alphabet, and ask students to group letters by topological equivalence. Note that this may differ according to the typeface used.

Main Activity cont...

Advanced

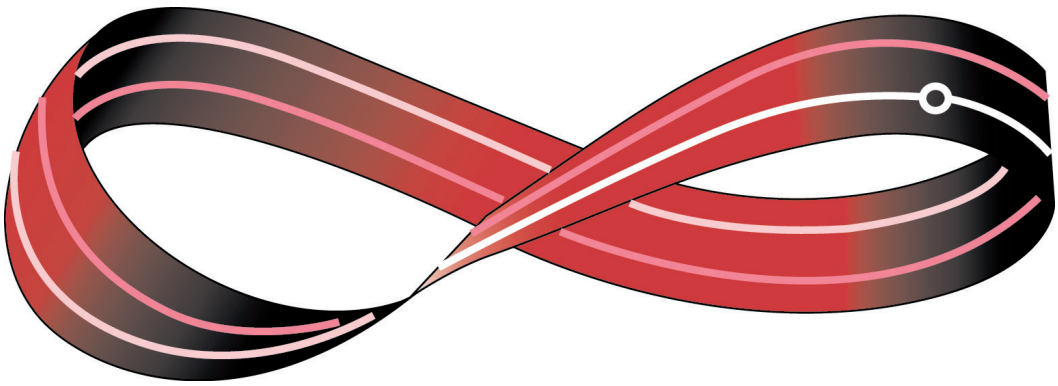
Ask students to consider the topological equivalence of the Platonic solids if (a) edges, faces and vertices are not preserved and (b) if edges, faces and vertices are. Then ask students to tabulate the number of faces, vertices and edges of the Platonic solids, and look for a relationship between them. Establish Euler's formula ($F+V=E+2$). Establish the duality of Platonic solids if a new solid is drawn by creating a new vertex at the centre of every face, and joining up each new vertex inside the original solid to new vertices on adjoining faces.

Extension Activity

Discuss what would constitute topological equivalence for classrooms within the school, and group classrooms accordingly.

Optional Extra

Get students to draw a topologically correct map of a part of the school and challenge their peers to identify which part of the school it is.



The Mobius band has only one edge and one surface.