Cartesian Coordinates

Key Learning Content

This film begins by considering what a 4-dimensional space would look like. Imagining the 4th dimension is difficult, and there is no simple answer to the question. The film encourages students to think about what we mean by dimensions, and in particular how 1-, 2-, and 3-dimensional spaces relate to each other. The use of coordinates to specify points in n-dimensional space is described. The number of vertices on a line, square and cube are counted, and a prediction is made for the number of vertices on a 4-dimensional cube, called a hypercube or tesseract. A representation of a hypercube is shown on screen.

Core Outcomes

Learning Points

- Be able to find subsequent terms of an integer sequence, e.g. 1, 2, 4, 8, and so on.
- Be able to understand the terms ‘face’, ‘edge’ and ‘vertex’.
- Be able to understand and use conventions for rectangular Cartesian Coordinates.
- Be able to determine the coordinates of the midpoint of a line segment given the coordinates of the two end-points.

Suggested Activities

- Generate sequences associated with solids and find formulas for the next term in the sequence.
- Classify 3-dimensional solids using Euler’s equation: $F+V=E+2$.
- Plot and locate points using 2- and 3-dimensional coordinates, then extend to four dimensions.

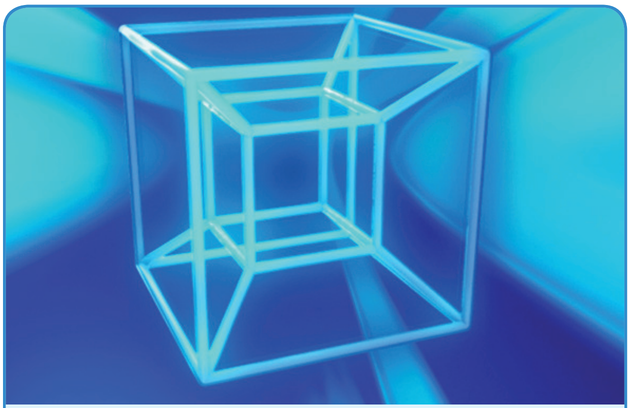
Extension Outcomes

Learning Points

- Be able to work out the dimension of a mathematical entity from thestructure of the formula that describes it.
- Be able to produce 2- and 3-dimensional representations of 4-dimensional objects.

Suggested Activities

- Take formulae for unknown mathematical entities and calculate the order of the formula by considering the power of terms it contains. Write down possible formulae for 4-dimensional entities.
- Visualise the shadow images of n-dimensional shapes on a 2-dimensional plane.
- Visualise cross-sectional cuts of n-dimensional shapes using a 2-dimensional plane.



A tesseract is a hypercube which has 16 vertices, 24 faces and 32 edges.

Related Films

To use before the lesson plan:

Painting By Numbers	This film shows how artists cope with the problem of rendering 3-dimensional space on a 2-dimensional canvas.
Vectors: Air Traffic Control	This film demonstrates how large numbers of planes avoid collisions at airports by using 3-dimensional vectors to manage take-offs and landings.
Polyhedra: Platonic Solids	This film provides a definitive analysis of 3-dimensional regular polyhedra.
Escher and the Endless Staircase	This film features an apparently logical but counterintuitive picture of an endless staircase in 3-dimensional space.

To use after the lesson plan:

The Seven Bridges of Konigsberg	This film relates the story of how an abstract study of vertices and edges solved a very practical problem.
The Biggest Number Ever	This film discusses how the largest number ever used in a mathematical proof happens to arise in a complex equation involving multidimensional cubes.

Guide Lesson Plan

Introduction

Begin by asking students what the 4th dimension is. The most common answer is likely to be ‘time’. Discuss other characteristics of our experience that could qualify as being an ‘extra dimension’ (e.g. smell, touch, colour, empathy). Then focus on what 4-dimensional space would look like.

Show Film

Cartesian Coordinates

Main Activity

Foundation

Revise how a point is plotted on a graph using 2-dimensional Cartesian Coordinates. Get students to visualise a 3-dimensional graph with (x,y,z) coordinates and locate points as best they can on a 3-dimensional graph. Test students’ understanding by asking them to calculate the coordinates of the midpoint of a line between two given points, in both 2- and 3-dimensional space. Then explore how these concepts could be extended to 4-dimensional space.

Main Activity cont ...

Advanced

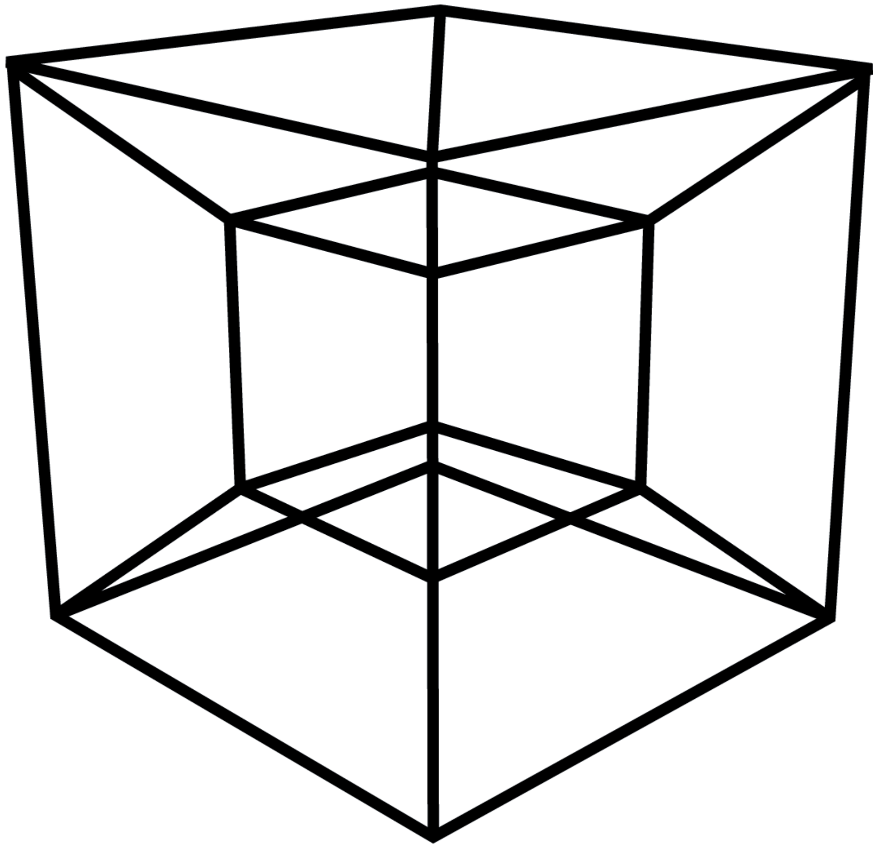
Hand out a list of formulas such as x^2yz^{-1} and $ab+b^2$ and explain how you can tell if they represent lengths, areas or volumes by considering the net power of each expression. Check students' understanding of the rule by using familiar formulas for areas/volumes of circles, spheres and cubes. Then ask: What would the formula for a 4-dimensional entity look like? Discuss what these entities might be called (e.g. hyper-area, hyper-volume).

Extension Activity

Commend students for being able to define points and write formulas for entities in 4-dimensional space, then invite them to visualise what they have defined. Imagine a horizontal line illuminated from the right side and casting a point shadow on a screen to the left; do the same for a square casting a line as its shadow. Extend for higher dimensions.

Optional Extra

Read the book, *Flatland: A Romance of Many Dimensions*, written in 1884 by English schoolmaster Edwin Abbott Abbott (or watch the film of the same name) for an entertaining description of life lived in different dimensional spaces.



Mathematicians use four coordinates w, x, y and z to describe a four-dimensional space.