Escher and the Endless Staircase

Key Learning Content

This film tells the story of the 'endless staircase' pictures created by the mathematician Sir Roger Penrose and the artist MC Escher. Other 'impossible shape' pictures are shown and explored. The pictures are used to illustrate the point that we do not simply see the world as it is, but interpret and make assumptions about what we are seeing. The film provides an opportunity to explore the idea of the impossible in mathematics, and question the assumptions we make about our knowledge.

Core Outcomes

Learning Points

- Be able to appreciate that there can be ambiguity in how we perceive objects.
- Be able to appreciate the role of perspective in creating optical illusions.

Suggested Activities

- Find examples of optical illusions.
- Categorise optical illusions by type.

Extension Outcomes

Learning Points

- Be able to understand the role of impossibility in mathematical proof.
- Be able to understand the meaning and applications of complex numbers.

Suggested Activities

- Work through examples of proof by contradiction.
- Carry out calculations involving complex numbers.

The artist Escher loved optical illusions and often used mathematical concepts in his art.

Related Films

To use before the lesson plan:

Painting By Numbers	This film shows how the geometry of perspective can create the illusion of 3D depth in 2D pictures.
Perspective: Parallax	This film explains how perspective can be used to calculate the distance of the furthermost stars.

To use after the lesson plan:

Topology	This film features geometry that focuses on curves and paths rather than straight lines and distances, creating mathematical shapes that challenge our powers of visualisation.
Tessellated Designs	This film examines a range of tessellations, from simple examples with regular polygons to the exotic tessellations of MC Escher.
Hyperbolic Geometry	This film gives alternative views to the Euclidian theory of how space is structured, as described by physicists and mathematicians such as Albert Einstein.
Irrational Numbers: Pythagoras	This film describes a discovery that challenged conventional beliefs and which had dire consequences for Greek mathematicians.
Imaginary Numbers	This film looks at the vast new field of mathematics that opens up once the square root of negative one is accepted as a number.

Guide Lesson Plan

Introduction

Ask students to think of examples of things which are impossible in mathematics. If necessary, prompt with: drawing a triangle with sides 3,4,8; solving $1/x = 0$, or $x^2=-1$. List examples and explore why we believe they are impossible.

Show Film

Escher and the Endless Staircase

Main Activity

Foundation

Give students access to the internet and get them to research Escher's other drawings, picking out their favourites and making a distinction between those which are 'impossible', like the staircase, and those that are simply surprising, like the Möbius strip. Gather and review selected images together.

Advanced

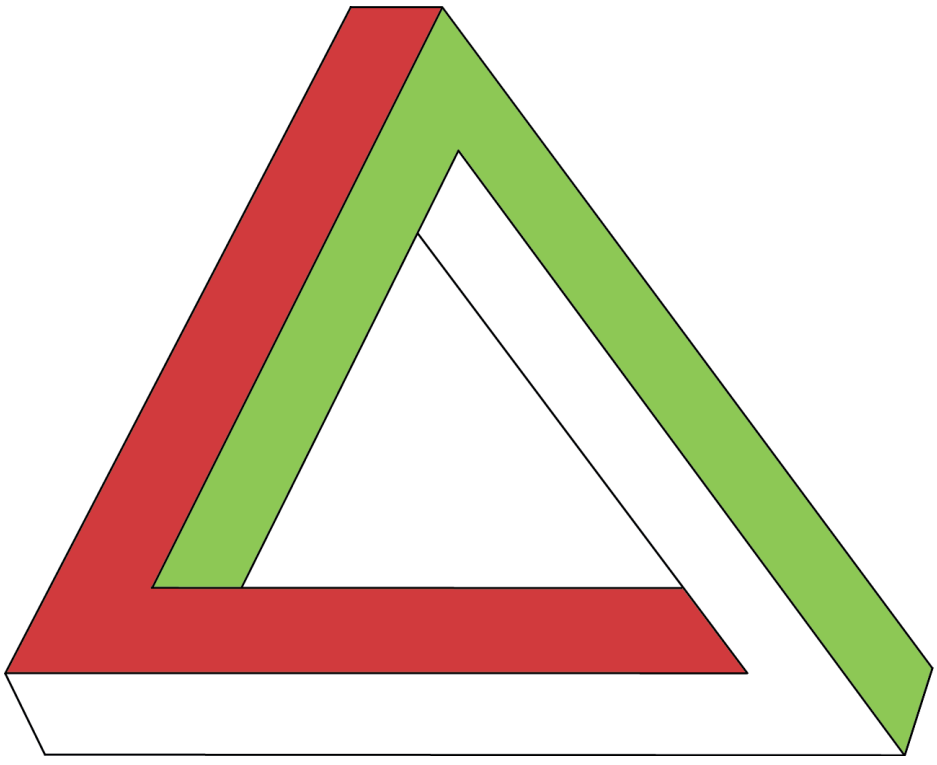
Give students access to the internet and get them to research the mathematical work of Sir Roger Penrose. What is twistor space? What is a Minkowski space? What do images of these spaces look like? How do these relate to what we can perceive? How might this explain Penrose's interest in the staircase? Get students to prepare brief presentations for their peers.

Extension Activity

Tell students that mathematicians have always been intrigued by what is considered impossible and sometimes this has led to new advances in mathematics. Ask them if they can solve $x^2=-1$ then give a brief introduction to complex numbers and complex number arithmetic.

Optional Extra

Explain how 'impossibility' is often used to prove results in mathematics. Go through proofs by contradiction of the irrationality of root 2, or the uncountability of real numbers.



The Penrose Triangle uses geometry to trick our sense of perception.