Proving Pythagoras

Key Learning Content

This film demonstrates the relationship between the sides of a right-angled triangle. The film explains that although the result was known before Pythagoras' time, the theorem is credited to him because he was the first to give a proof, using geometry. The result is given algebraically and geometrically. An algebraic proof of the theorem is given in detail, using results for the area of a square and right-angled triangle. The proof includes the result of squaring a bracket $(a+b)$. The film can be used to introduce geometry, algebra or trigonometry topics.

Core Outcomes

Learning Points

- Be able to understand the term 'right-angled triangle' and the properties of this triangle.
- Be able to understand and use Pythagoras' Theorem in two dimensions.

Suggested Activities

- Find the missing side of a right-angled triangle using Pythagoras' Theorem.
- Find the distance between points on a graph using Pythagoras' Theorem.
- Find the length of vectors given in i, j terms using Pythagoras' Theorem.

Pythagoras' Theorem concerns the dimensions of a right-angled triangle.

Extension Outcomes

Learning Points

- Be able to understand that symbols may be used to represent numbers in equations or variables in expressions and formulae.
- Be able to expand the product of two simple linear expressions.
- Be able to find the area of simple shapes using the formulae for the areas of triangles and rectangles.
- Be able to appreciate the distinction between geometric and algebraic proofs.

Suggested Activities

- Multiply out brackets and simplify results.
- Use the formula for the area of a triangle to prove sine and cosine rules.

Related Films

To use before the lesson plan:

Chinese Development of Maths	This film explains that Chinese mathematics developed independently from the West, yet included Pythagoras' result.
The Babylonians and Plimpton 322	This film contains an intriguing example of how Pythagorean triples appear to have been set for homework centuries before Pythagoras lived.
The Greeks and Proof	This film explores the Greek fascination with absolute certainty and proof in mathematics.

To use after the lesson plan:

Building the Pyramids	This film shows that the ancient Egyptians may have used right-angled triangles to ensure the accuracy of pyramid design.
The Arabic Science of Balancing	This film relates how Muslim mathematicians first used algebra in the middle of the 1st century AD.
Diophantine Equations: Fermat	This film gives an example of a Diophantine equation (finding three integer-value sides for a right-angled triangle) and discusses the work of the French mathematician, Fermat.

Guide Lesson Plan

Introduction

Show on screen the series of drawings made in Euclid's geometric proof of Pythagoras' Theorem, without mentioning the theorem. At the end, ask students what they have just seen.

Show Film

Proving Pythagoras

Main Activity

Foundation

Write up $a^2+b^2 = c^2$ on the board then set questions to find the missing side. Begin with finding the missing hypotenuse, then finding a missing right-angled side. Next, set questions in context, e.g. length between two points on a graph; length of a vector given as so many units across, so many up; length of a ladder propped up against a wall.

Main Activity cont ...

Advanced

Explain the history of algebra and the comparatively late development of algebraic proofs in mathematics. Revise the basics of algebraic manipulation with brackets, e.g. multiplying a bracket by a constant, multiplying two brackets. Revise standard results such as $(a+b)^2$, $(a-b)^2$, $(a+b)$ and $(a-b)$, then revise the algebraic proof of Pythagoras' Theorem as shown in the film. Next, extend to prove the cosine rule using Pythagoras and substitution. Observe that the cosine rule is just Pythagoras' Theorem when the angle is 90 degrees.

Extension Activity

Foundation

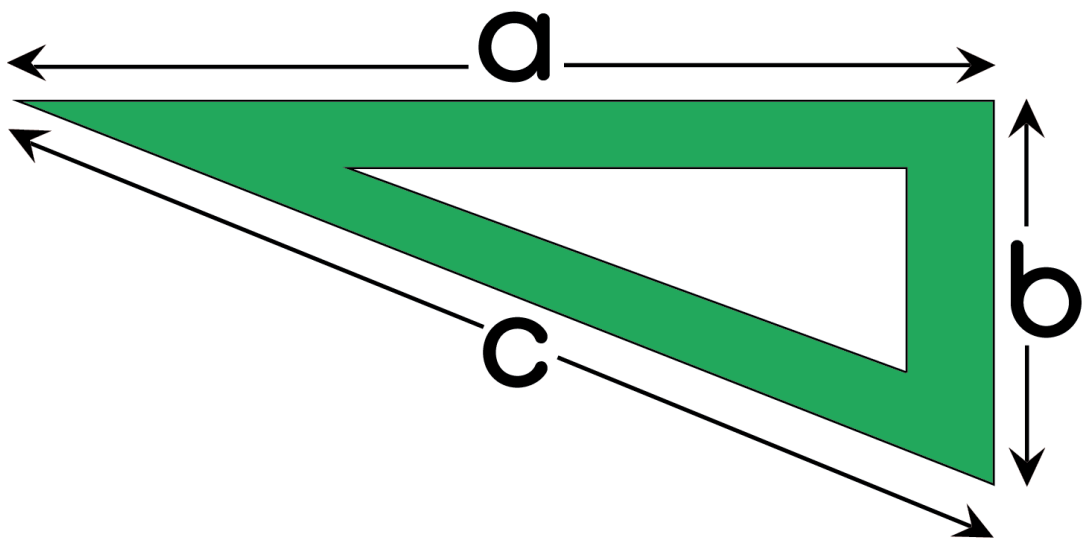
Explain what a Pythagorean triple is and ask students to find as many triples as they can. Distinguish between multiples of the same solution, and fundamentally different solutions.

Advanced

Use algebra to construct algebraic triples.

Optional Extra

There are many different proofs of Pythagoras's Theorem. Get students to research on the internet, find, learn and demonstrate other proofs to their peers.



$$a^2 + b^2 = c^2$$

The square of the longest side, the hypotenuse, is equal to the sum of the squares of the other two sides.