The Monty Hall Problem

Key Learning Content

This film describes a famous mathematical problem which featured in a popular American game show presented by Monty Hall. The game works thus: a contestant is offered a choice of three different doors, with a prize hidden behind only one of the doors; the contestant must try to select the door hiding the prize. The door chosen by the contestant is not opened but the host opens one of the other two doors. The dilemma for the contestant is as follows: if no prize is there, should they switch to the remaining door? The film gives the answer and explains in simple terms why it is correct, defining conditional probability along the way.

The teacher may wish to pause the film half way through, before the solution is given, and then return to it after some discussion.

Core Outcomes

Learning Points

- Be able to understand the language of probability in terms such as 'events', 'likelihood', 'random', 'dependency' and 'conditional probability'.
- Be able to list systematically all the outcomes for single events and for two successive events.
- Be able to recognise that $\Sigma P_i = 1$.
- Be able to draw and use tree diagrams.
- Be able to use simple conditional probability when combining events, e.g. picking two balls out of a bag, one after the other, without replacement.

Suggested Activities

- Run an experimental version of the Monty Hall problem and record success rates under switch and no-switch strategies.
- Come up with justifications for the switch strategy using tree diagrams or other methods.

Extension Outcomes

Learning Points

- Be able to understand and use estimates or measures of probability from theoretical models.
- Be able to understand and use the conditional probability formula and Bayes' Theorem.

Suggested Activities

- Use the conditional probability formula in simple problems.
- Derive Bayes' Theorem from the conditional probability formula.
- Justify the switch strategy in the Monty Hall problem using Bayes' Theorem.

In the game show Let's Make a Deal, switching always improved the player's chance of winning.

Related Films

To use before the lesson plan:

The Prisoners' Dilemma	This film provides a classic example of dependency between events.

To use after the lesson plan:

Logic: Bayesian Robots	This film explores the possibility of taking conditional probability to its extreme using artificial intelligence in robots as a context.
Probability: Irrational Fears	This film gives examples of situations where our initial instinctive reaction may not be the most appropriate.

Guide Lesson Plan

Introduction

Recreate the Monty Hall situation using a token prize under one of three cups. Choose a student to play the game in front of their peers. Repeat with other students, letting everyone discuss the player's strategy. Keep a tally of successes and failures under switch and no-switch strategies. Ask which strategy the students think is the right one, and why.

Show Film

The Monty Hall Problem

Main Activity

Foundation

Use the internet to find a Monty Hall app that allows the game to be played many times using different strategies; demonstrate experimentally that the switching strategy is on average better. Ask students how they would prove theoretically that this is always the case. Get students to come up with intuitively plausible justifications for the switching strategy using tree diagrams or other methods and predict the proportion of successes under either strategy. Check this theoretical prediction against actual experimental success rates.

Main Activity cont...

Advanced

Give the conditional probability formula and set simple problems to test its use. Then establish a relationship between p(A/B) and p(B/A) using the formula, and derive the result known as Bayes' Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Then justify the switch strategy using Bayes' Theorem.

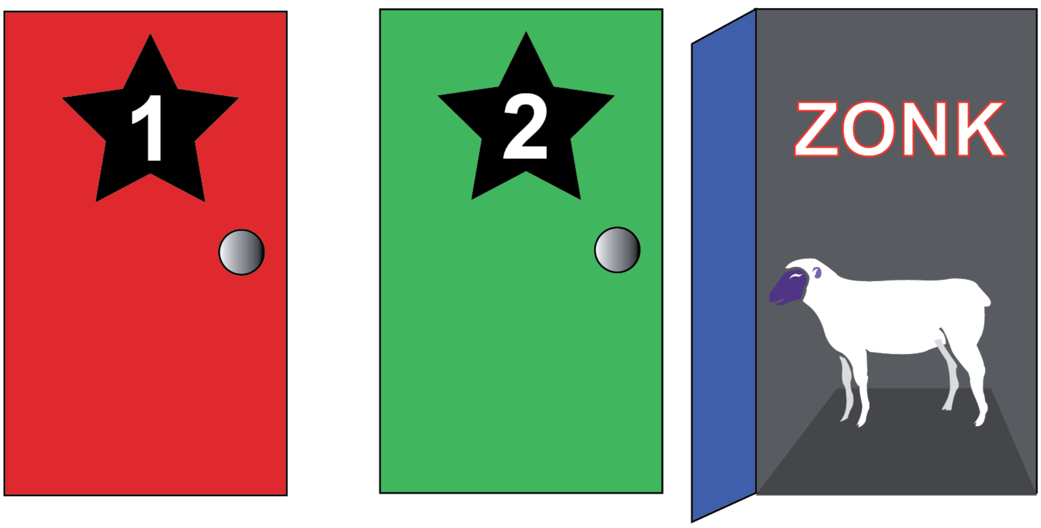
(e.g. Suppose door 1 is chosen. Then let the probability that the prize is behind door 2 be $p(Z_2)$; then $p(Z_2) = 1/3$. The probability that Monty will open door 3 = $p(O_3) = ½$. But then observe that $p(O_3|Z_2) = 1$. Use Bayes' Theorem to reverse the result and show that $p(Z_2|O_3) = 2/3$.)

Extension Activity

Consider how the Monty Hall problem changes if there are four doors instead of three, but all other aspects of the game are the same (i.e. the player chooses one door, one of the other losing doors is opened and the player is given the option to switch). Should the player still switch, and what are the relevant probabilities of success? Generalise the answer for even more doors.

Optional Extra

Use the internet to research the original Monty Hall problem and the audience reaction to it. What did academics of the time make of the problem? Did they always get the answer right? Pick a similar game show currently on TV and analyse the mathematics behind the choices it offers.



When the first door is chosen, the probability that door one holds the prize is 1 in 3, or 1/3.