## The Card Counter

## Key Learning Content

This film describes how a mathematician used complex mathematics to give him a more than $50 \%$ chance of winning at a game of cards. By counting cards the mathematician could work out the optimum time to place a bet on the game.

The difference between independent and dependent events is illustrated with reference to repeatedly rolling a die and drawing straws. The card game Blackjack is described in much detail.


## Core Outcomes

## Learning Points

- Be able to understand the language of probability in terms such as 'outcomes', 'events', 'dependent' and 'independent'.
- Be able to determine the probability that two or more independent events will both occur.
- Be able to use simple conditional probability when combining events, e.g. picking two balls out of a bag, one after the other, without replacement.


## Suggested Activities

- Solve probability problems involving two or more independent events.
- Solve probability problems involving two dependent events by listing possible outcomes.
- Work out probabilities for simple situations in the card games Blackjack or 21.


## Extension Outcomes

## Learning Points

- Be able to determine the probability that a sequence of dependent events will occur.
- Be able to understand and use estimates or measures of probability from theoretical models.
- Be able to estimate probabilities from previously collected data.


## Suggested Activities

- Work out the probability of dealing a given sequence of cards in order.
- Work out probabilities for complex situations in the card games Blackjack or 21 .


Dr Edward Thorpe unearthed an obscure formula said to reduce the odds of losing at Blackjack.

## Related Films

To use before the lesson plan:

## The Odds Are Against You

To use after the lesson plan:

## Why Do Shares Change Price?

## The Birthday Paradox

## Can You Trust Your IQ?

## How Algorithms Change the World

This film shows why the gambler rarely wins.

This film asks if probability can help explain the highly complex movements of share prices.

This film gives an application of dependent probabilities to work out whether two people have the same birthday.

This film explores the ability to remember the cards dealt in a game and asks whether this skill is a reliable indicator of intelligence.

This film explores the ubiquitous use of mathematics in modern day life, where what happens next depends on what has happened before.

## Guide Lesson Plan

## Introduction

Play a simple card game: take a card from a pack, show it to the students then ask them to guess if the next card drawn will be higher or lower. Repeat several times and get the students to keep a score of successful guesses. Ask the highest scorers what strategies they followed.

## Show Film 단

## The Card Counter

## Main Activity

## Foundation

Give formal definitions of dependent and independent events. Set simple problems involving independent events, e.g. What is the probability of rolling a one and then a two with two rolls of a die? Then move on to dependent probabilities, e.g. If six counters numbered one to six are placed in a bag, what is the chance of picking a one and then a two if the first counter is/is not replaced?

## Advanced

Ask students to work out the probabilities of dealing a given combination of cards in order, e.g. What is the probability of dealing an entire suit of cards in order from a shuffled deck of cards? Then extend to: What would the probability be of dealing an entire suit in any order? What would be the probability of dealing a hand of thirteen cards that did not contain any diamonds?

## Extension Activity

Explain the rules of Blackjack. Assume that a student has been dealt two cards which can be seen and the banker has two cards that cannot. Work out the probability that the next card dealt to the student will take their total over 21 for different combinations of their two cards.

## Optional Extra

What is the probability that three cards dealt from a shuffled pack will have a total of 21 or less? Test your theoretical answer by experiment.

|  | Blackjack |  |
| :---: | :---: | :---: |
|  | Total | Chances of |
|  | Cards | Busting |
|  | 21 | 100\% |
|  | 20 | 91\% |
|  | 19 | 85\% |
|  | 18 | 77\% |
|  | 17 | 69\% |
|  | 16 | 62\% |
|  | 15 | 58\% |
|  | 14 | 56\% |
|  | 13 | 39\% |
|  | 12 | 31\% |
|  | 11 | 0\% |

