Benford's Very Strange Law

Key Learning Content

This film tells the story of a strange and quite unexpected result discovered by the American physicist Frank Benford: that the distribution of first digits of naturally occurring numbers is far from random but follows a common pattern, with smaller numbers occurring far more frequently than larger numbers. First noticed in logarithmic tables, Benford's result has been shown to hold for river lengths, the magnitude of earthquakes and the distance of stars from the Earth. The use of Benford's Law in checking for falsified data is briefly described.

Core Outcomes

Learning Points

- Be able to understand the language of probability in terms such as 'outcomes', 'events', 'sample space', 'equal likelihood' and 'random'.
- Be able to understand and use the term 'expected frequency'.
- Be able to recognise that $\Sigma P_i = 1$.
- Be able to estimate probabilities from previously collected data.
- Be able to present data using bar charts.

Suggested Activities

- Take measurements of objects in the classroom then work out the distribution of first digits of the measurements.
- Take publicly available sets of data and work out the distribution of first digits.

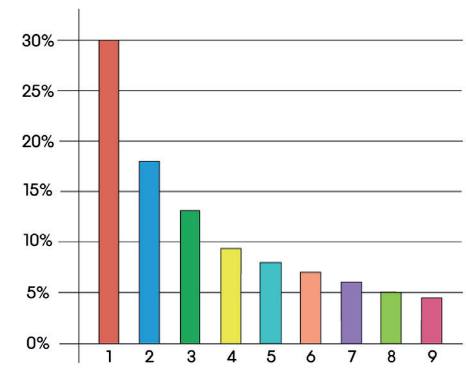
Extension Outcomes

Learning Points

- Be able to understand and use estimates or measures of probability from theoretical models.
- Be able to understand and use logarithms.
- Be able to apply probability to solve simple problems.

Suggested Activities

- Generate powers of 2, or factorials, and work out the distribution of first digits.
- Analyse the logarithms of integers 1 through 10 and relate their distribution to Benford's Law.



Surprisingly, Benford's Law appears in data from the natural world.

Related Films

To use before the lesson plan:

Logic: Bayesian Robots	This film shows how robots can be helped to learn, using Bayesian statistics, by looking for patterns and making sense of experience.

To use after the lesson plan:

The Richter Scale	This film explains that the magnitude and frequency of earthquakes follows a logarithmic pattern.
A Pattern in the Primes	This film tells the story of the ongoing search by mathematicians to find a pattern in the frequency of prime numbers.

Guide Lesson Plan

Introduction

Ask students what they understand by the word 'random'. Use a random number generator on a spreadsheet or scientific calculator to generate random numbers between 1 and 10.Check that the first digits are evenly spread over the numbers 1 through 9. Ask students what they would expect to find if they looked at the first digits of naturally occurring numbers in the real world. Agree that most people would expect these to be random.

Show Film

Benford's Very Strange Law

Main Activity

Teachers may wish to carry out these activities before showing the film in order to avoid any unintentional bias:

Foundation
Get students to measure items in the classroom with a tape measure (e.g. the length and width of a book, a desk, a window, a door, a computer screen, a pencil, or an eraser) and record the measurements on the board. Calculate the frequency of first digits 1 through 9 and compare results with Benford's Law. Repeat with a different unit of measurement (e.g. feet and inches vs. centimetres and metres) and check that the result still roughly holds.

Advanced
Get students to generate a list of powers of 2, or factorials or integers, and calculate the frequency of first digits 1 through 9. Compare the results with Benford's Law. Repeat with powers of larger numbers and check that the result still holds.

Extension Activity

Foundation

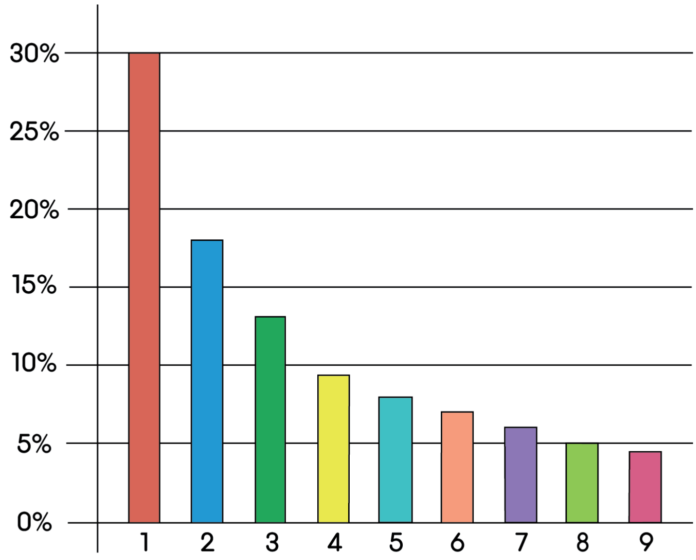
Find publicly available lists of measurements, such as the lengths of major rivers in the world, or the world records for athletic events, and calculate the frequency of first digits 1 through 9. Confirm that these roughly conform to Benford's Law.

Advanced

Explain what a logarithm is, then calculate the logarithms (to base 10) of numbers 1 through 10. By considering the differences between the log values, come up with an explanation of Benford's Law. (Hint: the differences between the log values are very close, expressed as percentages, to the frequencies Benford observed.)

Optional Extra

Ask students to imagine that a pattern – as yet unknown – did exist in the frequencies of first digits in measurements, independent of the unit of measurement being used. Consider what would then happen if 'metres' were replaced by 'half-metres', so that measurements between 1 and 2 became measurements between 2 and 4, measurements between 2 and 4 became measurements between 4 and 8, and so on. Does this help to explain why Benford's Law holds true?



Benford found that first digit use falls in sequence: there is a 30% chance that number 1 will be the first digit compared to only a 4% chance that 9 will be the first digit.