Logic: Bayesian Robots

Key Learning Content

This film shows robots working with the help of Bayesian statistics, a type of statistics that uses Bayes' Theorem for conditional probability. A robotic hospital porter negotiates obstacles in a corridor; another robot finds and brings a stapler. Bayes' Theorem is shown on screen. Its application is illustrated with an example working out the probability that a patient has measles given that they have spots.

Students will need a thorough understanding of basic probability before viewing this film. Knowledge of Set Theory would also be helpful.

Core Outcomes

Learning Points

- Be able to understand that symbols may be used to represent numbers in equations or variables in expressions and formulas.
- Be able to understand the language of probability, in terms such as 'conditional probability' and 'the probability of A given B'.
- Be able to understand and use the formula for conditional probability.

Suggested Activities

- Illustrate the conditional probability formula using simple examples and apply it to simple problems.
- Illustrate the measles/spots relationship shown in the film using Venn diagrams.

Extension Outcomes

Learning Points

- Be able to understand and use Bayes' Theorem for dependent events.
- Be able to understand and use estimates or measures of probability from theoretical models.
- Be able to estimate probabilities from previously collected data.

Suggested Activities

- Show how Bayes' Theorem can be derived from the conditional probability formula.
- Use Bayes' Theorem to work out the probabilities of events based on experimental evidence.

Bayes' Theorem is a result in probability theory.

Related Films

To use before the lesson plan:

The Monty Hall Problem

This film explores a famous game show dilemma that can be resolved using Bayes' Theorem.

To use after the lesson plan:

Benford's Very Strange Law

This film shows how the distribution of first digits of naturally occurring numbers is far from random but follows a common pattern, with smaller numbers occurring far more frequently than larger numbers.

Variables: Dating By Numbers

This film looks at the use of quantitative and qualitative data to predict the likelihood of attraction between men and women.

How Algorithms Change the World

This film gives examples of the use of algorithms to drive automated processes, from life support machines to share trading.

Guide Lesson Plan

Introduction

Describe the following scenario to the group: you offer to toss a coin and let them off homework if it lands tails-up. You then toss the coin one, two, three times and every time it comes up heads. Ask students what they think the probability is that the coin is biased? Then say that you toss the coin again, a dozen times, and again it comes up heads every time. What would they say now about the probability that it is biased? What if this happened a hundred times? Explain that mathematicians have developed a formal theory that calculates a precise probability that the coin is biased, taking into account all the experimental evidence that is available.

Show Film

Logic: Bayesian Robots

Main Activity

Foundation

Write down the conditional probability formula and explain what each part of the formula means. Apply the formula in simple situations, e.g. what is the probability that you get a 4 when you roll a die, given that the number you get is even? Draw a Venn diagram and explain that each area of the Venn diagram can be considered as a probability. Then show and explain the formula:

$$p(AB) = p(A) + p(B) – p(AB)$$

by thinking of adding together areas and subtracting the double-counted intersection. Then return to the conditional probability formula and justify it by considering areas of the Venn diagram ('given B' means that the only bit of the diagram that matters is B).

Main Activity cont ...

Advanced

Write down the conditional probability formula and show how Bayes' Theorem can be derived from it:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Explain that the formula is used to 'update' an estimate of an unknown probability p(A) to p(A|B), where B is 'new data'. This updating process can be applied repeatedly, allowing the robots in the film to learn from the data they collect.

Apply the formula to the measles (M) and spots (S) case with starting probabilities p(M) = 1/1000 and p(S) = 1/2000, and with p(S/M) = 1 (you always get spots if you have measles). Show that p(M/S) = ½.

Extension Activity

Foundation

State Bayes' Theorem as above, and run through the same measles/spots calculation. Then illustrate the result by considering a population of 200,000 people and probabilities as above: 100 will have measles, 200 will have spots. Assume all people with measles have spots. Draw a Venn diagram for this data with M as a subset of S. Point out that 100 of the 200 people with spots have measles, so we say p(M|S) = 100/200 = ½, as the formula stated.

Advanced

Consider the double-headed coin example described at the beginning of the lesson. Suppose the Central Bank tells you that one in a million coins in circulation is double-headed. Suppose you see a coin tossed four times and it come up heads each time. What is the probability that the coin is double-headed (DH)? Your first answer, before the coin is tossed, is p(DH) = 1/1,000,000. Using Bayes' Theorem, after getting four heads (4H):

$$p(DH \mid 4H) = \frac{p(DH)}{p(4H)} \times p(4H \mid DH)$$

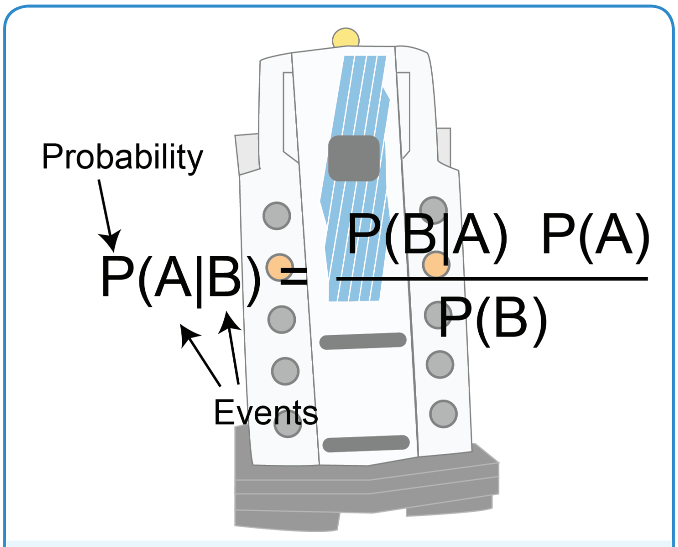
Now p(4H) = $(½)^4$ = 1/16 and p(4H|DH) = 1, so

$$p(DH \mid 4H) = \frac{1/1000000}{1/16} \times 1 = \frac{16}{1000000}$$

Extend for five or more heads in a row.

Optional Extra

Extend the above approach to more complex problems, e.g. suppose you know that one in one thousand dice is biased, so that the probability of getting a six is 1/3. Suppose you roll a die ten times and get a 6 ten times. What is the probability that the die is biased? Consider how this sort of reasoning can be applied in industry, for example with possibly faulty electronic components on a satellite.



Bayes' Theorem is a way of understanding how the probability of the theory being true is affected by new evidence.