The Egyptians and Unit Fractions

Key Learning Content

This film relates the story of the Egyptian bird god, Horus, parts of whose eye were used to denote unit fractions – fractions with one as their numerator. The Egyptians first used fractions to divide up land, split the year into seasons and calculate taxes.

Unit fractions are defined and examples given of expressing any modern fraction as the sum of unit fractions. The special place given to unit fractions in antiquity explains why we use the term 'vulgar fraction' today. Familiarity with fractions and fraction arithmetic would be useful prior to watching the film.

Core Outcomes

Learning Points

- Be able to identify common denominators.
- Be able to use common denominators to add and subtract fractions.
- Be able to understand and use mixed numbers and vulgar fractions.

Suggested Activities

- Practise addition and subtraction of fractions with different denominators.
- Express non-unit fractions as sums of unit fractions.
- Order fractions by expressing them over a common denominator.

Extension Outcomes

Learning Points

- Be able to understand and use unit fractions as multiplicative inverses.
- Be able to use index notation and index laws for multiplication and division of positive integer powers.

Suggested Activities

- Practise multiplication and division of fractions with different denominators.
- Write the fractions on the Eye of Horus as powers of two.

Parts of the Eye of Horus were used to denote fractions.

Related Films

To use before the lesson plan:

Building the Pyramids	This film speculates about the type of mathematics the Egyptians may have used in building the pyramids.
The Egyptians and Multiplication	This film describes an ingenious method of long multiplication based on successive doubling of numbers.

To use after the lesson plan:

Fractions: Pythagorean Tuning	This film explains how the Greeks used fractions to make music.
Binary: What Is Binary?	This film helps to make the link suggested in the learning materials between using unit fractions and writing numbers in binary form.

Guide Lesson Plan

Introduction

Ask students when they think people first started using fractions as we use them today. Explain that the Egyptians, Greeks and Romans would have made no sense of 2/3 or 3/7, but that they did have unit fractions.

Show Film

The Egyptians and Unit Fractions

Main Activity

Foundation

Recap rules for addition and subtraction of fractions. Then ask students how they would write ¾ or 5/6 using only unit fractions. Suggest that they could write 7/13 as seven lots of 1/13 added together; then ask: Is there a simpler way of doing this? Ask what ½ + 1/26 equals. Then set out a method for finding the shortest way of writing any fraction using only unit fractions:

How would $\frac{5}{7}$ be written?

Start by taking away the largest unitary fraction you can: - $\frac{1}{2}$

So $\frac{5}{7}-\frac{1}{2}=\frac{10-7}{14}=\frac{3}{14}$

Then do the same again with $\frac{3}{14}$ Take away the largest unitary fraction you can: - $\frac{1}{5}$

So $\frac{3}{14}-\frac{1}{5}=\frac{15-14}{70}=\frac{1}{70}$

Hence $\frac{5}{7}=\frac{1}{2}+\frac{1}{5}+\frac{1}{70}$

Check ($\frac{1}{2}+\frac{1}{5}+\frac{1}{70}=\frac{35+14+1}{70}=\frac{50}{70}=\frac{5}{7}$)

Set students more fractions to write in their simplest unitary form.

Advanced

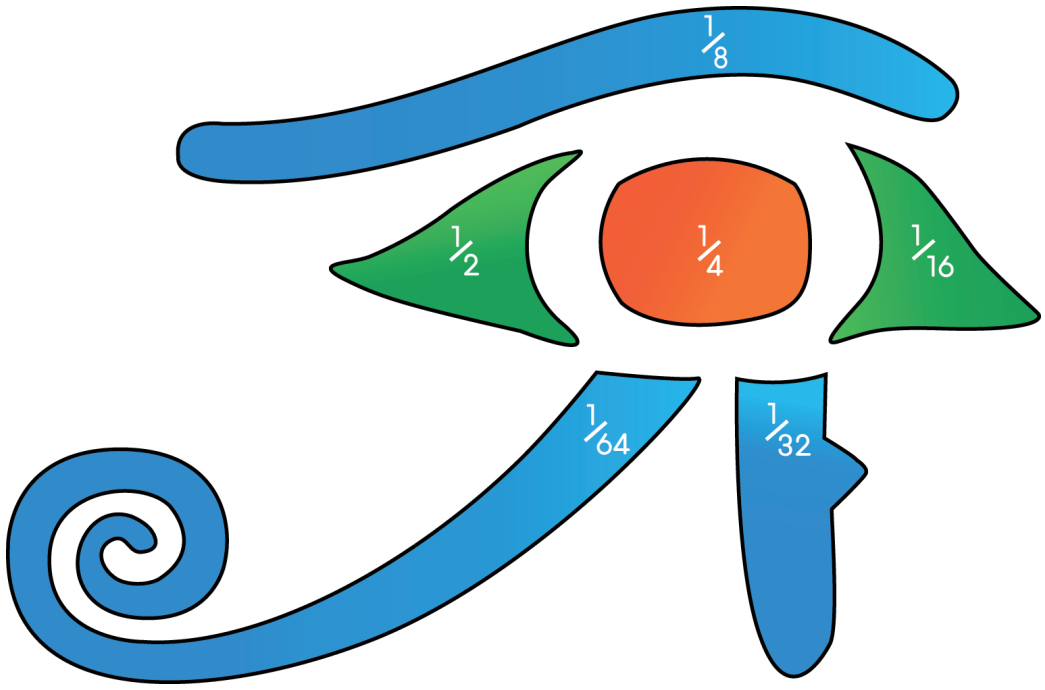
Look at the fractions in the Eye of Horus and ask students what is special about them. Get them to write the fractions as powers of 2 (either $1/2^n$ or 2^{-n} depending on their familiarity with indices). Ask students whether or not it would be possible to express any fraction between zero and one using only these fractions. Give examples and set questions to do in class. Then ask students, what would the sum of all these fractions be, if you carried on adding successive negative powers of 2? Show how to prove the sum to infinity is 1.

Extension Activity

Give students different modern fractions close to the same value, e.g. 1/3, 32/97, 153/460 and ask them to order the fractions. Then get them to convert the fractions into their simplest expression using only unit fractions (using the method above). Then look again at ordering the fractions. Can they see any advantage to writing fractions using only unit fractions?

Optional Extra

Look at the unit fractions in the Eye of Horus and continue the sequence, with powers of two in the denominator. Ask students, what is the connection between these fractions and the binary system? How does this help to answer the question, can all fractions between 0 and 1 be written as unit fractions using only powers of 2?



The Egyptian system was based on unit fractions, which always have one on the top, and a whole number on the bottom.