The Greeks and Proof

Key Learning Content

This film begins by describing the attitude of the ancient Greeks to knowledge, as something that lay within an immortal soul which had to be discovered through deductive reasoning. Axioms and theorems are defined, and the process of proof described, but students are not required to follow a mathematical proof on screen. Examples of Greek mathematical theorems are given. Euclid's role in collecting together and standardising the presentation of proofs is mentioned. The film ends with the existence of as yet unproven mathematical conjectures.

Core Outcomes

Learning Points

- Be able to understand the language of mathematical proof in terms such as 'axiom', 'theorem', 'deductive reasoning', and 'logic'.
- Be able to understand the nature of mathematical proof and its link to logical reasoning.
- Be able to recognise Euclid's axiomatic method in geometry.

Suggested Activities

- Work through Plato's construction of a square twice the area of a given square.
- Demonstrate and use the Intersecting Chord Theorem.

The ancient Greeks developed axioms, or statements based on universally accepted self-evident truths.

Extension Outcomes

Learning Points

- Be able to construct simple geometrical proofs using the properties of triangles and circles.
- Be able to understand the meaning of surds, and manipulate surds in problem solving.

Suggested Activities

- Prove the Angle in a Semicircle Theorem from first principles.
- Prove the Angle at the Centre Theorem from first principles, then prove the Angles in Same Segment and the Angle in a Semicircle Theorems.
- Show how to prove the irrationality of root 2.
- Apply vector methods for simple geometrical proofs.

Related Films

To use before the lesson plan:

Proving Pythagoras

This film looks at the famous theorem which is known by generations of students – but is it true?

To use after the lesson plan:

How Origami Changed the World

This film discusses a simple problem posed by the ancient Greeks which was solved thousands of years later by the Japanese art of paper folding.

Geometry: Euclid

This film describes the life and work of the famous Greek mathematician.

Irrational Numbers: Pythagoras

This film tells the story of what happened when the ancient Greeks realised that root 2 cannot be written as a fraction.

Where is the Centre of a Triangle?

This film considers what is meant by the centre of a triangle and shows that there are many different interpretations of, and answers to, this deceptively simple question.

Designing Chartres

This film shows how the ancient Greeks' knowledge of circles helped to build one of the most beautiful cathedrals in the world.

Guide Lesson Plan

Introduction

Ask the students what they know for certain. Get them to write down in a list things they know that are absolutely, unambiguously 'true'. Then show them your list of mathematical statements, from 1 + 1 = 2, to Pythagoras' Theorem. Of all the statements listed by the students, discuss which are the 'most true' (i.e. most beyond doubt). Ask students how they know their statements are true.

Show Film

The Greeks and Proof

Main Activity

Foundation

Set students the task originally described in Plato's famous work, Meno, where a slave boy is asked to construct a square with twice the area of a given square. Get students to work on this in pairs and then present their answers to their peers. Then get a couple of students to read out Plato's original text, and demonstrate the solution (i.e. rotate the square 45 degrees then add triangles to each corner to construct a larger square which will then have twice the area of the smaller). Discuss what the Greeks meant when they said we all had knowledge locked within us. Then move on to one of the results proven by the Greeks known as the Intersecting Chord Theorem, for chords drawn within a circle. Explain the theorem, then get students to check it works by drawing circles and chords and checking the maths. Ask: How can we prove the theorem is true?

Advanced

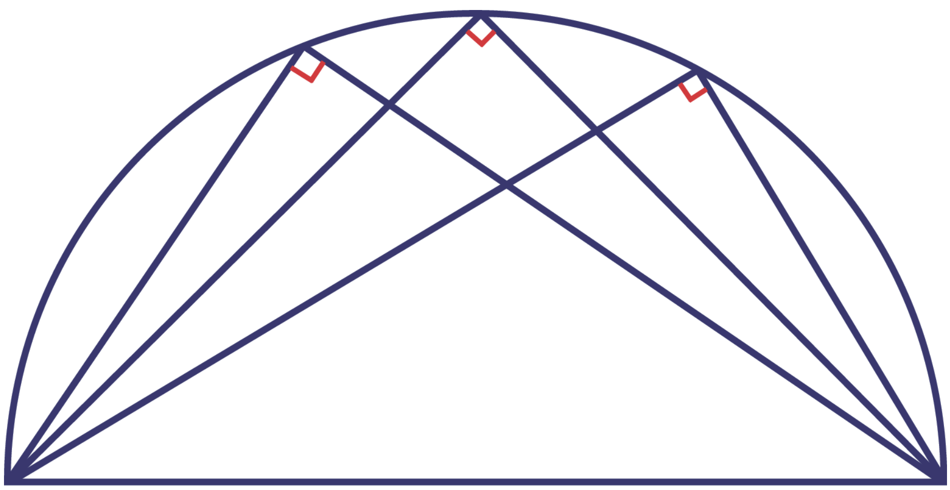
Go over the statement of the Angle in a Semicircle Theorem shown in the film. Get students to draw triangles in semicircles and demonstrate the result. Then ask: How would you prove that it is always true? By drawing a radius from the centre of the circle to the vertex of the enclosed triangle, and considering angles within the two triangles so formed, prove the theorem. Ask students if they can prove the Intersecting Chord Theorem discussed above.

Extension Activity

State and/or prove from first principles the Angle at the Centre Theorem, then get students to prove the Angles in Same Segment and the Angle in a Semicircle Theorems from this result.

Optional Extra

Introduce vectors and show how they can be used to prove general statements about triangles, e.g. prove that for any triangle, the line joining the midpoints of two sides is always parallel to and half the length of the third side.



One of Thales' first proofs used logic to deduce that any triangle inscribed in any semicircle will always be a right-angled triangle.