

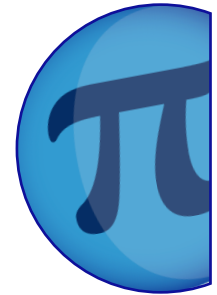


# The Greeks and Proof

NAME: .....

CLASS: .....

DATE: .....



## Basic

1) State if the following are: always false, sometimes true, or always true:

a) Tomorrow is Friday.

b) If you double a whole number you get an even number.

c) If you multiply two odd numbers you get an odd number.

d) There are four weeks in a month.

e) If you add one to an even number you get an even number.

f) If the 3rd of September is a Friday, the 11th of September will be Saturday.

g) If you can divide the last two digits of a year exactly by four, it will be a leap year.

h) February always has fewer days than November.



# The Greeks and Proof

## Basic

2) State if the following are true or false:

- a) The sum of the angles in a triangle is  $360^\circ$ .
- b) If a triangle has three equal sides, then it will have three equal angles.
- c) The sum of the angles at a point is  $360^\circ$ .
- d) If the four angles in a quadrilateral are  $90^\circ$ , then it's a square.
- e) The sum of the angles on a line is  $180^\circ$ .
- f) A square is a rectangle.
- g) The diameter of a circle is twice the radius.
- h) A rhombus is a parallelogram.

3) Decide if the following mathematical statements are true or false for all numbers  $x$  and  $y$ :

- a)  $x + y = y + x$
- b)  $x - y = y - x$
- c)  $xy = yx$
- d)  $\frac{x}{y} = \frac{y}{x}$
- e) For any number  $x$ ,  $x^2 > 0$
- f)  $(xy)^2 = x^2y^2$
- g)  $x(x + 1) = x^2 + 1$
- h) If  $x$ ,  $y$  and  $z$  are even, then  $x + y + z$  is even

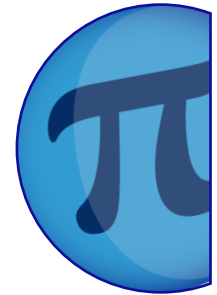


# The Greeks and Proof

NAME: .....

CLASS: .....

DATE: .....



## Core

1) State if the following are true or false:

- a) The sum of the angles in a triangle is  $360^\circ$ .
- b) If a triangle has three equal sides, then it will have three equal angles.
- c) The sum of the angles at a point is  $360^\circ$ .
- d) If the four angles in a quadrilateral are  $90^\circ$ , then it is a square.
- e) The sum of the angles on a line is  $180^\circ$ .
- f) A square is a rectangle.
- g) The diameter of a circle is twice the radius.
- h) A rhombus is a parallelogram.

2) Decide if the following mathematical statements are true or false for all numbers  $x$  and  $y$ :

- a)  $x + y = y + x$
- b)  $x - y = y - x$
- c)  $xy = yx$
- d)  $\frac{x}{y} = \frac{y}{x}$
- e) For any number  $x$ ,  $x^2 > 0$
- f)  $(xy)^2 = x^2y^2$
- g)  $x(x + 1) = x^2 + 1$
- h) If  $x$ ,  $y$  and  $z$  are even, then  $x + y + z$  is even



# The Greeks and Proof

---

## Core

- 3) a) Take any three consecutive even numbers and add them together. What do you notice?
- b) Show that the sum of any three consecutive even numbers is always a multiple of 6.
- 4) a) Take any three consecutive even numbers and multiply them together. What do you notice?
- b) Show that the product of any three consecutive even numbers is always a multiple of 8.

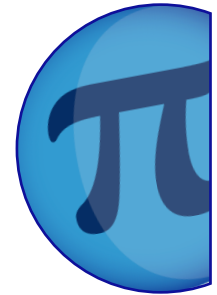


# The Greeks and Proof

NAME: .....

CLASS: .....

DATE: .....



## Advanced

1) Decide if the following mathematical statements are true or false for all numbers  $x$  and  $y$ :

a)  $x + y = y + x$

b)  $x - y = y - x$

c)  $xy = yx$

d)  $\frac{x}{y} = \frac{y}{x}$

e) For any number  $x$ ,  $x^2 > 0$

f)  $(xy)^2 = x^2y^2$

g)  $x(x + 1) = x^2 + 1$

h) If  $x$ ,  $y$  and  $z$  are even, then  $x + y + z$  is even

2) a) Take any three consecutive even numbers and add them together. What do you notice?

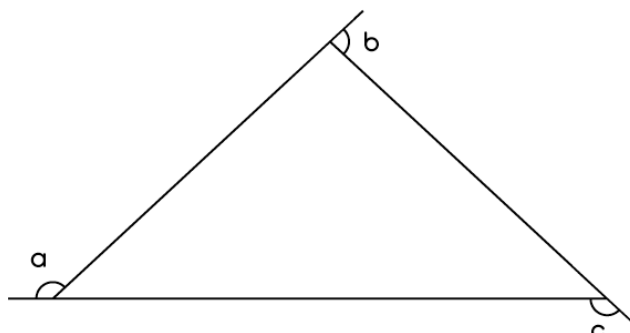
b) Show that the sum of any three consecutive even numbers is always a multiple of 6.

3) a) Take any three consecutive even numbers and multiply them together. What do you notice?

b) Show that the product of any three consecutive even numbers is always a multiple of 8.

4) The diagram opposite is formed from three straight lines.

Prove that  $a + b + c = 360^\circ$





# The Greeks and Proof

## ANSWERS

### Basic

- 1) a) sometimes true      b) always true      c) always true      d) always true  
e) always false      f) always true      g) sometimes true      h) always true

- 2) a) False      b) True      c) True      d) False  
e) True      f) True      g) True      h) True

- 3) a) True      b) False      c) True      d) False  
e) False      f) True      g) False      h) True

### Core

- 1) a) False      b) True      c) True      d) False  
e) True      f) True      g) True      h) True

- 2) a) True      b) False      c) True      d) False  
e) False      f) True      g) False      h) True

3) a) Multiple of 6.

b) The first number is a multiple of 2, therefore we can write it as  $2N$ , where  $N$  is a whole number. The three consecutive even numbers can now be written as:  $2N + (2N + 2) + (2N + 4)$ .

Adding these numbers gives  $6N + 6$  which is 6 times  $(N + 1)$ . Therefore as it is 6 times, the total will always be a multiple of 6.

4) a) Multiple of 8.

b) As in the above example, the three numbers can be written as  $2N$ ,  $(2N + 2)$  and  $(2N + 4)$ . Multiplying the numbers:

$$2N \times (2N + 2) \times (2N + 4)$$

$$= 2N \times 2(N + 1) \times 2(N + 2)$$

$$= 2 \times 2 \times 2 \times N \times (N + 1) \times (N + 2)$$

$$= 8N(N + 1)(N + 2)$$

Which is 8 times  $N(N + 1)(N + 2)$ . Therefore as it is 8 times, the product must be a multiple of 8.



# The Greeks and Proof

## ANSWERS

### Advanced

- |            |          |          |          |
|------------|----------|----------|----------|
| 1) a) True | b) False | c) True  | d) False |
| e) False   | f) True  | g) False | h) True  |

2) a) Multiple of 6.

b) The first number is a multiple of 2, therefore we can write it as  $2N$ , where  $N$  is a whole number. The three consecutive even numbers can now be written as:  $2N + (2N + 2) + (2N + 4)$ .

Adding these numbers gives  $6N + 6$  which is 6 times  $(N + 1)$ . Therefore as it is 6 times, the total will always be a multiple of 6.

3) a) Multiple of 8.

b) As in the above example, the three numbers can be written as  $2N$ ,  $(2N + 2)$  and  $(2N + 4)$ . Multiplying the numbers:

$$\begin{aligned} & 2N \times (2N + 2) \times (2N + 4) \\ &= 2N \times 2(N + 1) \times 2(N + 2) \\ &= 2 \times 2 \times 2 \times N \times (N + 1) \times (N + 2) \\ &= 8N(N + 1)(N + 2) \end{aligned}$$

Which is 8 times  $N(N + 1)(N + 2)$ . Therefore as it is 8 times, the product must be a multiple of 8.

4) Angles inside the triangle will be  $(180 - a)$ ,  $(180 - b)$  and  $(180 - c)$ . Angles in a triangle total to  $180^\circ$ .

$$\text{Therefore } (180 - a) + (180 - b) + (180 - c) = 180$$

$$540 - (a + b + c) = 180$$

$$-(a + b + c) = -360$$

$$a + b + c = 360$$