Irrational Numbers: Pythagoras

Key Learning Content

This film tells the story of a lesser known part of Pythagoras' work: his quest to explain the world with rational numbers, or fractions, alone. When he and his followers discovered that the square root of two could not be written as a fraction using integers, their fundamental beliefs about the world were challenged.

Irrational numbers are defined, and other examples given. Familiarity with fractions and integers would be useful prior to watching the film.

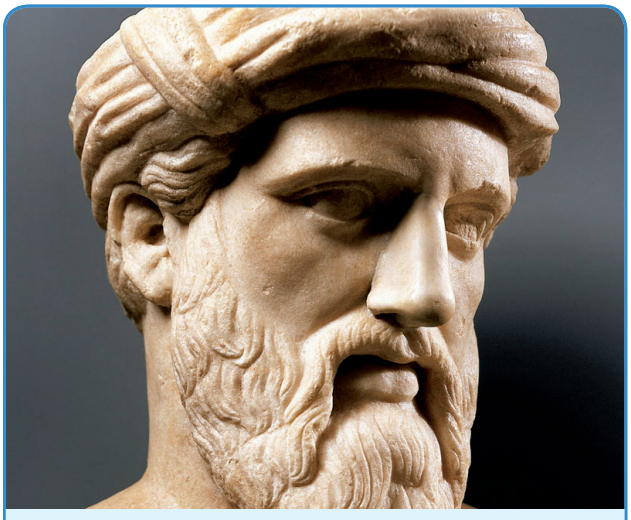
Core Outcomes

Learning Points

- Be able to understand the meaning of rational and irrational numbers, and recognise examples of each.
- Be able to understand the meaning of surds.
- Be able to understand and use Pythagoras' Theorem in two dimensions.

Suggested Activities

- Take a list of numbers and sort them into rational and irrational.
- Find rational approximations to irrational numbers.



Pythagoras is credited with discovering many of the mathematical concepts that now define our world.

Extension Outcomes

Learning Points

- Be able to recognise that terminating and recurring decimals can be written as fractions a/b, where a and b are integers.
- Be able to recognise that non-recurring, non-terminating decimals cannot be written as fractions a/b, where a and b are integers.
- Be able to manipulate surds, including rationalising the denominator where the denominator is a pure surd.
- Be able to understand and use proof by contradiction.

Suggested Activities

- Convert fractions into decimals, and finite or recurring decimals to fractions.
- Take a list of expressions containing surds and simplify them to identify those that are the same.
- Prove the irrationality of root two using proof by contradiction.

Related Films

To use before the lesson plan:

The Prime Number Code	This film describes the special pattern that may exist between prime numbers.

To use after the lesson plan:

Proving Pythagoras	This film looks at the theorem most often associated with Pythagoras' name, concerning the hypotenuse of a right-angled triangle.
Fractions: Pythagorean Tuning	This film demonstrates how the ancient Greeks explained musical harmony through fractions.
Calculating Pi: Archimedes	This film explores the best-known irrational number, which is essential for the mathematics of circles.
The History of the Golden Ratio	This film shows how another irrational number crops up in science and nature.

Guide Lesson Plan

Introduction

Ask students to use their calculators to work out the square root of two, giving their answer to as many decimal places as possible. Then show the decimal expansion of root two to 40 or 50 places (from the internet), and ask: How would you know if the expansion ever stopped or recurred?

Show Film

Irrational Numbers: Pythagoras

Main Activity

Foundation

Begin by going over the terminology, giving the students definitions of rational and irrational numbers; finite, recurring and non-recurring decimals; and surds. Check students' understanding of the terms by giving them numbers and asking what type of number each is. Establish the correspondence between irrational numbers and non-recurring, infinite decimals. Ask the students how many irrational numbers they can list. Explain that if root two is irrational, then so are the surds +1, +2,+3 etc.

Main Activity cont ...

Advanced

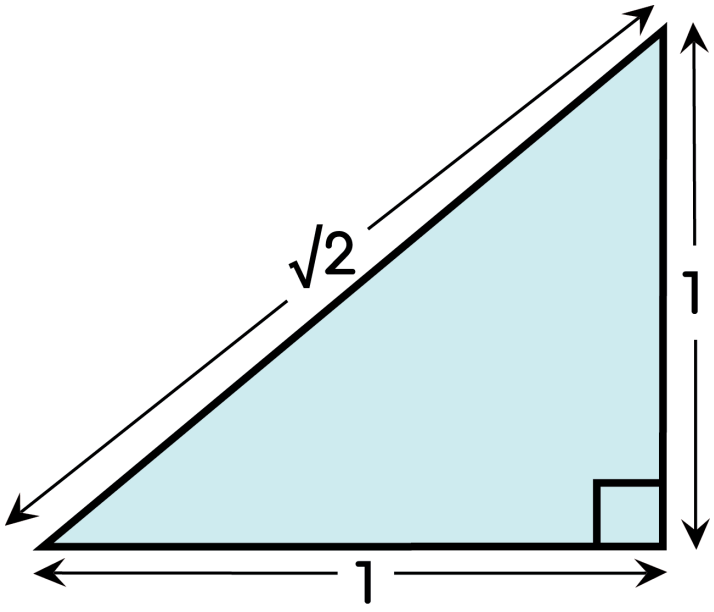
Show how to write any fraction as a (possibly recurring) decimal using long division. Then demonstrate how to convert a recurring decimal into a fraction by multiplying through by a suitable power of 10 and subtracting the original number. Generalise the method for a recurring decimal of any length. Ask students what the irrationality of root two means for its decimal expansion.

Extension Activity

Run through the proof of the irrationality of root two using proof by contradiction. Talk through the advantages and disadvantages of this method of proof. Can the students prove the irrationality of root three by a similar method?

Optional Extra

Expressions containing irrational roots are called surds. Surds can be added, subtracted, multiplied or divided just like any other numbers. Research the arithmetic of surds and give examples of these operations.



Irrational numbers cannot be expressed as a fraction a/b, where a and b are integers and b is not zero.