Calculating Pi: Archimedes

Key Learning Content

This film tells how the Greek mathematician, Archimedes, came up with a remarkably accurate estimate of the value of Pi.

Pi is defined as the ratio of a circle's circumference to its diameter. The Egyptian estimate of Pi = root (10) is given, a value close to 3.16. Archimedes' estimate using the area of polygons constructed inside and outside a circle is then described in some detail. Inequality signs are used to express his estimate. The film ends with a reference to the irrational nature of Pi.

Core Outcomes

Learning Points

- Be able to find circumferences and areas of circles using relevant formulae.
- Be able to recognise the terms 'centre', 'radius', 'diameter', 'circumference', 'chord', 'arc' and 'sector' of a circle.
- Be able to find the perimeter of shapes made from triangles and rectangles.
- Be able to understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle.
- Be able to identify upper and lower bounds where values are given to a degree of accuracy.
- Be able to understand and use the symbols >, <, ≥ and ≤.
- Be able to understand and use mixed numbers and vulgar fractions.

Suggested Activities

- Calculate the area of n-sided regular polygons inscribed inside and outside a circle and hence estimate the value of Pi.
- Calculate the perimeter of n-sided regular polygons inscribed inside and outside a circle and hence estimate the value of Pi.
- Find progressively more accurate fraction approximations to the value of Pi.
- Use the formulas for area and circumference of a circle to solve simple problems.

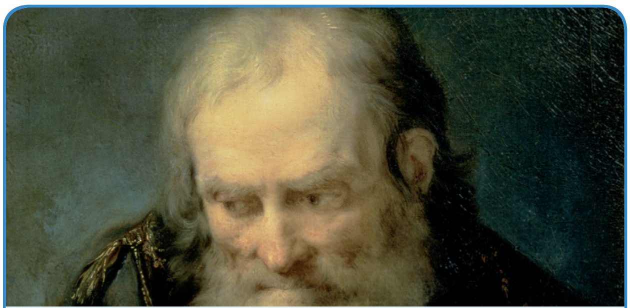
Extension Outcomes

Learning Points

- Be able to recognise that a terminating decimal is a fraction.
- Be able to appreciate the links between finite, recurring and non-recurring decimals, and rational and irrational numbers.
- Be able to convert recurring decimals into fractions.
- Be able to provide reasons, using standard geometrical statements, to support numerical values for angles obtained in any geometrical context involving lines, polygons and circles.

Suggested Activities

- Derive the formulas for the area and perimeter of an n-sided regular polygon used in Archimedes' approximations.
- Show that any recurring decimal can be written as a fraction.



Archimedes' study of circles gave the world its best approximation of one of the most famous mathematical constants.

Related Films

To use before the lesson plan:

Designing Chartres

This film shows how circles and their sophisticated properties were used in the design of the beautiful French cathedral of Chartres.

To use after the lesson plan:

Pi: Reciting Pi

This film explores the infinite and non-recurring nature of the decimal expansion of Pi and the challenge of trying to remember it.

Chinese Development of Maths

This film describes how Chinese mathematics developed independently from the West and included the calculation of Pi correct to seven decimal places.

Spirals in Nature

This film looks at occurrences in nature of many forms of spiral, including the Archimedes spiral.

Guide Lesson Plan

Introduction

Show the trick of cutting a circle up into many equal sectors, then placing the sectors side by side, alternately pointing up and down, to form a rough rectangular shape. Demonstrate that if the circumference of the circle is known to be 2.Pi.r ($2\pi r$), then the rectangle area (and hence the area of the circle) can be estimated as Pi.r x r = Pi.r^2 (πr^2). Then ask: how would you work out the value of Pi?

Show Film

Calculating Pi: Archimedes

Main Activity

Foundation

Give students the formula for the area of a regular n-sided polygon in terms of n, the number of sides. Get them to calculate (using a calculator) the area of regular polygons inscribed in/ circumscribed around a circle of radius 1 and hence estimate Pi. Work out how accurate their estimates are as a percentage of Pi. Ask students to think about the problems Archimedes would have had in doing these calculations without an electronic calculator.

Main Activity cont ...

Advanced

Using trigonometry, derive the formulas for the area and perimeter of a regular n-sided polygon in terms of n, the number of sides. Use these formulas to work out estimates of Pi within given bounds for large values of n (n=100, 200, 1000...).

Extension Activity

Foundation

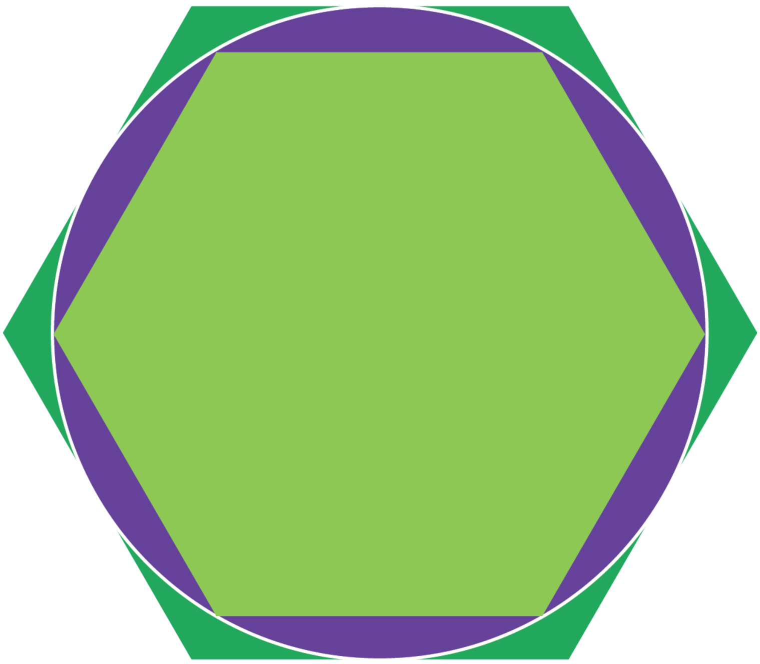
Repeat the estimation process using the circumferences of the inscribed and circumscribed polygons, instead of the area. Which approach (area or circumference) gives the most accurate estimates, for any given value of n?

Advanced

Take the decimal expansion of Pi and find fractions in their lowest terms that approximate to Pi, starting with 22/7. Show that any recurring decimal can be written as a fraction. If Pi does not recur, what can you conclude?

Optional Extra

Why is Pi for area the same as Pi for circumference? Does this logically have to be the case? Could the two numbers differ at, say, the millionth decimal place? Research how mathematicians prove that Pi for area is exactly the same value as Pi for circumference.



To calculate the area of a circle, Archimedes drew a circle within two straight-edged shapes. He then calculated the area of both shapes to determine that the circle's area lay somewhere between the two.