Diophantine Equations: Fermat

Key Learning Content

This film tells the story of Fermat's Last Theorem: that there are no integer solutions to the equation $x^n + y^n = z^n$ when n is an integer greater than 2. The theorem is described by Andrew Wiles, the mathematician who finally proved Fermat's theorem more than three centuries after it was proposed. Fermat's equation is an example of a Diophantine equation, i.e. one for which the solutions are integers, or whole numbers.

Core Outcomes

Learning Points

- Be able to understand and use integers.
- Be able to calculate squares and cubes.
- Be able to understand that symbols may be used to represent numbers in equations or variables in expressions and formulae.
- Be able to use correct notational conventions for algebraic expressions and formulae.
- Be able to evaluate expressions by substituting numerical values for letters.

Suggested Activities

- Find solutions to the equation $x^2 + y^2 = z^2$.
- Show that there are no solutions to the equation $x^3 + y^3 = i^3$ for positive integer values of x, y, z up to 20.

Extension Outcomes

Learning Points

- Be able to use index notation for negative integer powers.
- Be able to substitute positive and negative integers in expressions and formulae.
- Be able to find general algebraic solutions to equations.

Suggested Activities

- Find a general algebraic solution to $x^2 + y^2 = z^2$.
- Explore whether there are solutions to the equation $x^n + y^n = z^n$ for negative integer values of n.

Fermat's Last Theorem was a simple Diophantine equation.

Related Films

To use before the lesson plan:

The Chase

This film gives an introduction to an example of simultaneous equations, using footage of a lion catching its prey to illustrate the concept.

To use after the lesson plan:

Number Theory: Gauss

This film describes the work of Gauss, a mathematician famous for his work in Number Theory.

The Babylonians and Plimpton 322

This film provides an overview of Babylonian mathematics, including their work on Pythagorean triples.

Proofs: Million-Dollar Maths

This film features some difficult mathematical problems, the solutions to which have eluded mathematicians for many years and continue to do so.

Proving Pythagoras

This film demonstrates a proof of Fermat's result when $n = 2$ and x, y, z are the sides of a right-angled triangle.

Guide Lesson Plan

Introduction

State Fermat's Last Theorem and tell the students they would be famous if they could find a simple proof or counter-example. Discuss what would count as a mathematical proof, and what a counter-example would look like.

Show Film

Diophantine Equations: Fermat

Main Activity

Foundation

Explain what a Diophantine equation is, then get students to find integer solutions for x and y of the linear equations $ax + by = c$ where a, b, c are integers. Explain that Fermat's equation is a Diophantine equation with powers of x, y and z, and that it does have solutions for n=2. Then ask students to find as many solutions for $n = 2$ as they can (3, 4, 5 and 5, 12, 13 are given in the film). Explain that these solutions are called Pythagorean triples.

Main Activity cont ...

Advanced

Show how any particular solution to the equation $x^2 + y^2 = z^2$ can generate further solutions simply by multiplying the solutions by a common multiplier.

A more difficult task is to find solutions that are not simply multiples of each other. Set students the challenge of finding the general (algebraic) form of the solution to $x^2 + y^2 = z^2$ by considering $z = m^2 + n^2$ and $x = m^2 - n^2$, for integers m, n where $m > n$.

Extension Activity

Get students to show that there are no solutions to Fermat's equations for $n = 3$ and values of x, y and z up to 20; and for $n = 4$ and values of x, y and z up to 10. Explore if the equation holds for negative powers.

Optional Extra

Are there any integer solutions to the equation $x^n + y^n = z^m$, where n and m can differ? Find integer solutions for x, y, z when $n = 2$ and $m = 3$. (Example: 22 + 112 = 53.)

$$3^2 + 4^2 = 5^2$$

$$5^2 + 12^2 = 13^2$$

$$x^n + y^n = z^n$$

Fermat's theory is featured in the Guinness Book of Records as the world's most difficult maths problem.