Set Theory: Cantor

Key Learning Content

This film tells the story of the sometimes troubled life of Georg Cantor and his groundbreaking work on Set Theory. The basic ideas of Set Theory are illustrated using the idea of an orchestra as a set of musicians. Subsets, the number of elements of a set, and the empty set are defined. The ideas of equal and equivalent sets are introduced. The deeper use of Set Theory within mathematics to define numbers and, in particular, infinite numbers is mentioned. The film can be used as a very broad introduction to Set Theory, making links to more complex ideas that are covered in related films.

Core Outcomes

Learning Points

- Be able to understand the definition of a set of numbers.
- Be able to understand the concept of the null set and its symbols Ø or { }.
- Be able to understand when two are sets are equivalent.

Suggested Activities

- List all members of a finite set given a simple definition.
- Determine whether something is a member of a (finite or infinite) set.
- Describe sets mathematically given their members.

Extension Outcomes

Learning Points

- Be able to understand and use subsets: if A is a subset of B, then $A \subset B$.
- Be able to use the notation n(A) for the number of elements in the set A.
- Be able to understand that there are different types of infinity.

Suggested Activities

- List all subsets of a given finite set.
- Determine whether one set is a subset of another.
- Determine the number of elements in a given set.

Set Theory involves the study of collections of numbers or objects. An orchestra can be seen as a set, but it also contains subsets such as strings.

Related Films

To use before the lesson plan:

Numbers: Life Without Numbers

This film explores the argument that the idea of sets or collections of objects is more basic than the idea of numbers, and asks whether we need numbers at all.

To use after the lesson plan:

Venn Diagrams: Global Habitats

This film introduces Venn diagrams and basic set notation such as ∪ and ∩.

Sets: Infinity

This film shows how Set Theory can be used to understand different types of infinity.

Guide Lesson Plan

Introduction

Show mathematical statements written in set language and ask students if they can interpret them, e.g.

Let A = {x: x is a student in this class}

and B = {x: x has handed in their homework}

then n(B) < n(A)

(Answer: 'Not everyone has handed in their homework')

Show Film

Set Theory: Cantor

Main Activity

Foundation

Begin with simple verbal definitions of sets, e.g. the set of all four-sided shapes, and get students to list their members.
Introduce formal set notation of the form

A = {x: x satisfies a given condition}

and get students to list members of a set given a definition, and define a set given its members.

Introduce the empty set and get students to define sets that are empty, e.g.

A = {x: x is an odd multiple of 4}.

Introduce notation for set membership and get students to determine whether something is a member of a given set, e.g. is 2,367,231 an member of the set

A = {x: x is a multiple of 9}?

Main Activity cont ...

Advanced

Define basic set and subset notation then get students to work out whether one set is a subset of another. For small sets, get students to list all the subsets of a set, including the set itself and the empty set. Introduce n(A) notation and get students to find a relationship between the number of elements in a finite set and the number of subsets of the set.

Extension Activity

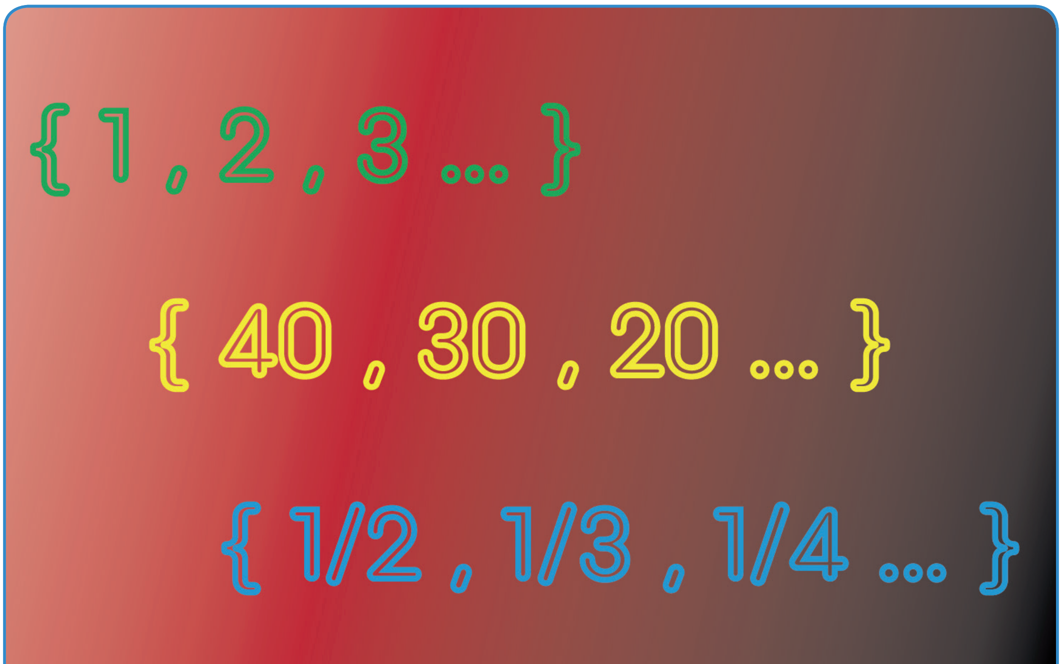
Replace verbal definitions of sets with algebraic definitions, e.g.

$$A = \{x: x/3 \quad \} \mathbb{N}$$

and show how this is equivalent to saying that A is the set of all multiples of 3. Set problems using algebraic notation, and get students to write solutions in algebraic form.

Optional Extra

Ask students to consider sets that may or may not be members of themselves. For example, the set of collections of objects is itself a collection of objects and thus is a member of itself, whereas the set of animals is not an animal, so it is not a member of itself. Ask students to research “the set of all things that are not members of themselves” and explain why this leads to a fundamental problem of logic. Why is this set so important in the history of mathematics?



Cantor’s work on Set Theory focused on the concept of infinity, through sets of numbers that carry on forever.