Coordinate Geometry: Descartes

Key Learning Content

This film tells the story of the life of René Descartes, the French mathematician and philosopher responsible for the (x,y) coordinate system used extensively in modern mathematics. By combining algebra with geometry, he transformed the way mathematicians thought about lines, curves and shapes. Examples of points described by coordinates on a graph are shown on screen, as well as equations for straight lines and quadratic curves.

Core Outcomes

Learning Points

- Be able to understand and use conventions for rectangular Cartesian Coordinates.
- Be able to locate points with given coordinates.
- Be able to plot points (x, y) in any of the four quadrants of a graph.
- Be able to determine the coordinates of points identified by geometrical information.

Suggested Activities

- Plot given points on a graph to create shapes and patterns.
- Decode messages given in terms of coordinates for letters placed on a graph.

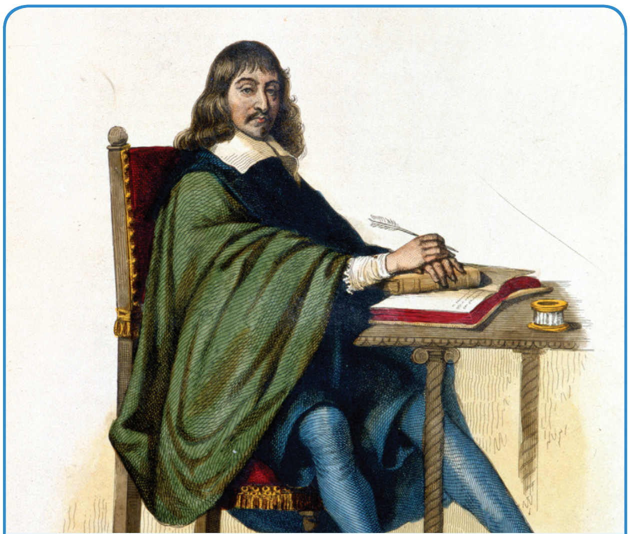
Extension Outcomes

Learning Points

- Be able to recognise that equations of the form $y = mx + c$ are straight line graphs.
- Be able to generate points and plot graphs of linear and quadratic functions.

Suggested Activities

- Pair up straight lines on a graph with equations in the form $y = mx + c$.
- Draw up tables of points (x,y) for given equations and then plot these on a graph to generate curves.



Descartes' theory stated that any point of a geometric shape could be found using an algebraic formula.

Related Films

To use before the lesson plan:

Geometry: Euclid

This film describes how Descartes' coordinate-based approach to curves revolutionised the study of geometry set out two thousand years earlier by Euclid.

To use after the lesson plan:

Vectors: Air Traffic Control

This film features a very practical use of coordinates in three dimensions.

Arches

This film looks at how architectural shapes can be described using Descartes' mathematics.

Straight Lines: Bee Lines

This film speculates as to whether bees use Cartesian Coordinates when working out the shortest distance between a flower and the hive.

The Mirror Lines of the Taj Mahal

This film uses Cartesian Coordinates to analyse the shape of one of the most famous buildings in the world.

Can Eating Fish Prevent Murder?

This film demonstrates how plotting x against y on a graph can suggest the causes of murder.

Guide Lesson Plan

Introduction

If possible, find a model cone which comes apart to reveal circles, ellipses, parabolas and hyperbolas (or use images of sections of a cone from the internet). Explain that people thought about curves like this, as conic sections, before the 16th century.

Show Film

Coordinate Geometry: Descartes

Main Activity

Foundation

Set exercises where students plot points on a graph then join the points to create recognisable shapes. Next, place letters on a graph and write coded messages where each letter in the message is given by the coordinates of the letter on the graph. Finally, get students to draw outlines of shapes on a graph and then describe the outlines by giving the coordinates of points on the outline.

Main Activity cont ...

Advanced

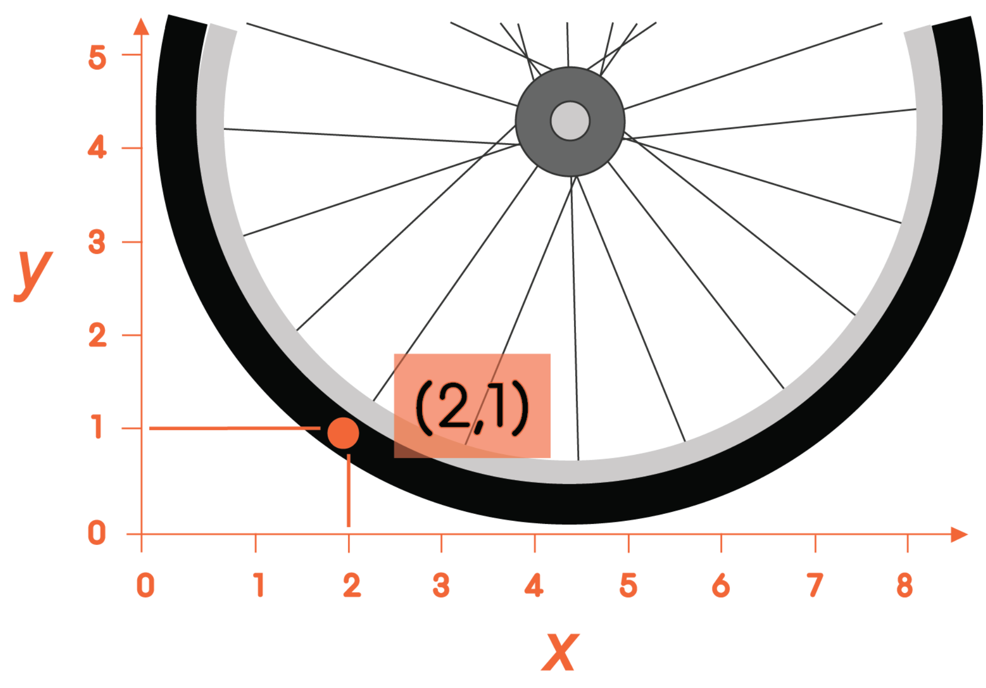
Give equations of lines in the form $y = mx + c$ and get students to generate points (x,y) satisfying the equations, then plot them on a graph. Look for patterns linking the values of m and c in the equations with the slope and positions of the resulting lines on the graphs. Then show students a graph with many lines drawn on it, and get them to pair up each line with given equations by considering gradients and y-intercepts.

Extension Activity

Show how the equation $y = x^2 + 5$ generates a U-shaped curve on a graph centred around the y-axis and with its lowest point at (0,5). Explain that this is a quadratic equation and that the curve is a parabola. Link this to the conic section shown at the start of the lesson. Plot different curves and explore how the curves change as the equation changes. Do the same for hyperbolas, cubics and other curves.

Optional Extra

Ask students to research the solution to $x^2 + y^2 = 1$. Say that the answer is a geometric shape, not an algebraic term. Then ask: What would happen to the equation if the centre was (a,b) and the radius 10?



In two dimensions each point can be described using the values denoted by letters x and y: one to describe the horizontal position, and the other to describe the vertical position. As this point moves, the values of the coordinates change.