Calculus: Newton

Key Learning Content

This film tells the story of the life of the English physicist and mathematician, Sir Isaac Newton. Newton studied the dynamics of the natural world, from the movement of a horse and cart to the orbits of the planets.

Differential and integral calculus are introduced. Examples of the application of calculus to distance-time and speed-time graphs are shown. Newton's discovery of gravity is mentioned. No knowledge of calculus is assumed in the film, although a thorough understanding of algebra and coordinate geometry would be useful prior to watching the film.

Core Outcomes

Learning Points

- Be able to find the gradients of non-linear graphs by drawing a tangent.
- Be able to understand the concept of a variable rate of change.
- Be able to interpret information presented in distance/time and speed/time graphs.
- Be able to understand and use the relationship between average speed, distance and time.

Suggested Activities

- Plot quadratic curves on a graph and find the gradient at a given point by construction.
- Solve problems involving distance, speed and time using distance/time and speed/time graphs.

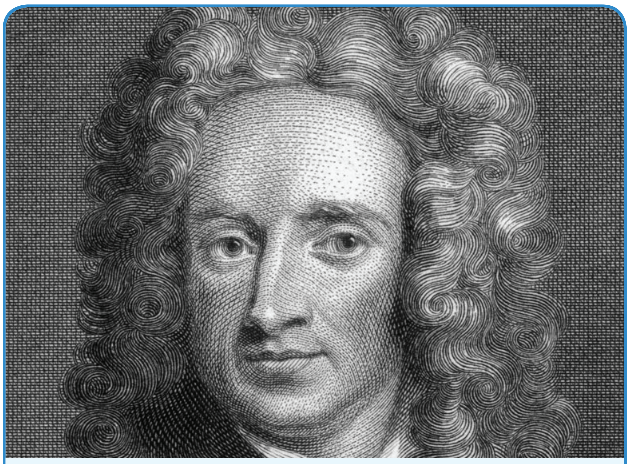
Extension Outcomes

Learning Points

- Be able to understand the rules of differentiation and integration and how they relate to each other.
- Be able to differentiate and integrate integer powers of x.
- Be able to determine gradients, rates of change, turning points (maxima and minima) by differentiation and relate these to graphs.

Suggested Activities

- Plot quadratic curves on a graph and find the gradient at a given point by differentiation.
- Solve problems involving distance, speed and time using calculus.



To describe these systems of change, Newton invented calculus.

Related Films

To use before the lesson plan:

Gradients: Fold Mountains — This film explains the nature and development of fold mountains and discusses rates of change in their gradients.

To use after the lesson plan:

Spirals in Nature — This film examines complex curves found in nature that can be described using mathematics.

The Chase — This film shows how equations can be used to analyse the speed of predator and prey and to predict the outcome of a chase.

Aiming for the Outer Planets — This film demonstrates the use of Newton's law of gravity to send probes into deep space.

Chaos By Mistake — This film illustrates the enormous problems in using mathematics to predict everyday events.

Guide Lesson Plan

Introduction

Throw a ball up in the air and watch it fall to the ground. Ask students to sketch on a graph (a) its distance from the ground and (b) its speed as time changes. How long did it take to reach the ground? Discuss and compare the students' results.

Show Film

Calculus: Newton

Main Activity

Foundation

Explain that for an object dropped from rest, the distance it has travelled in time t is given approximately by the formula:

$$D = 5t^2$$

Get the students to plot a graph of distance against time. Explain that speed is the rate of change of distance with respect to time, or the gradient of the graph. By drawing tangents, find the speed of the falling object at different times. Then plot a graph of speed (calculated from the tangents) against time, and observe that the points will lie roughly on a straight line. Explain that the gradient of this line is the rate of acceleration due to gravity and calculate this. ($g = 10\ m/s^2$)

Main Activity cont ...

Advanced

Explain that differentiation is an algebraic method of calculating the rate of change. Give standard differentiation results and practise differentiating simple polynomial expressions. Get students to draw a quadratic graph and calculate the gradient at several points by drawing tangents. Then check the tangent results using differentiation. Show that if, for a falling object,

$$\text{distance} = D = 5t^2$$

then differentiation gives

$$\text{speed} = dD/dt = 10t.$$

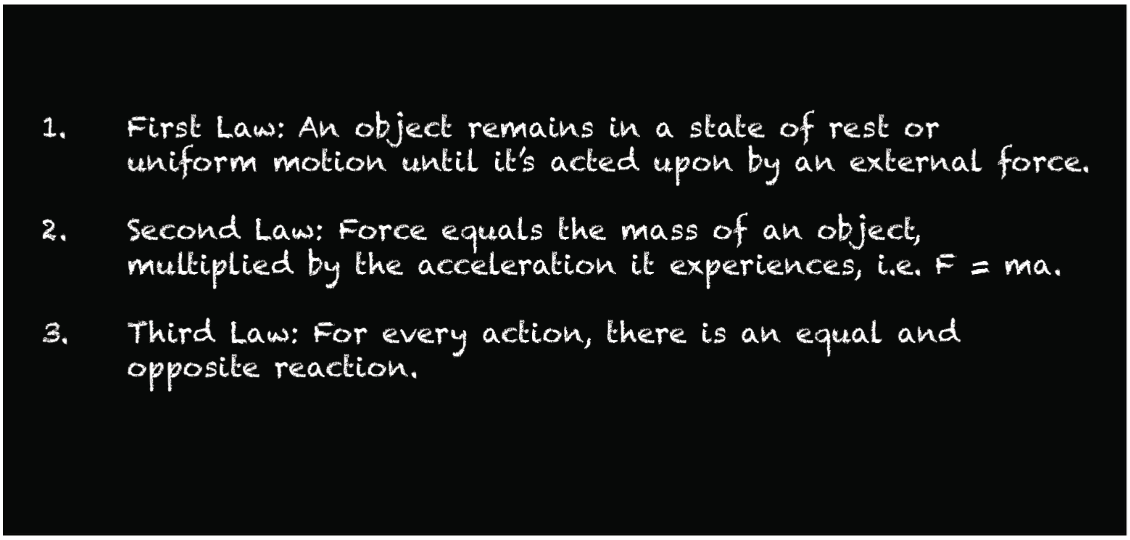
Ask students how they would then show that acceleration due to gravity is approximately 10 m/s^2.

Extension Activity

Draw graphs/write equations for distance, speed and acceleration versus time for an object thrown up, reaching maximum height, and then falling down. Discuss what must be true about the object's speed at maximum height. What is its acceleration throughout this motion?

Optional Extra

Get students to research Newton's work, and find out what his Law of Universal Gravitation is. What are the variables used in the equation for gravity? Describe in words the relationships between them.



Newton applied calculus to his study of planetary orbits and acceleration, which led him to develop his Laws of Motion.