The Biggest Number Ever

Key Learning Content

This film describes Graham's Number, a number which, when it was found, was the largest number ever used in a mathematical proof. Professor Ron Graham discovered the number in 1977, when working on a problem involving multidimensional cubes.

The film begins by describing much smaller numbers: a googol, and a googolplex, both defined in terms of powers of 10. The concept of a 'power tower' is mentioned and shown on screen. Graham's Number is described as a limit, an upper boundary to the solution of a problem. The number itself is then shown on screen using the Knuth up-arrow notation. Graham is then shown working with modular arithmetic to prove that the last digit of his number is seven.

The film should be used to explore the idea of very large numbers, and to show how they can be written mathematically. Understanding the complexity of googols, power-towers, limits or modular arithmetic is not necessary to understand the film.

Core Outcomes

Learning Points

- Be able to recognise and use the names of large numbers, from one thousand to one googol.
- Be able to express numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$.
- Be able to use index notation and index laws for multiplication and division of positive integer powers.

Suggested Activities

- List names of numbers known by students and fill in the gaps for powers of ten to 10^{99}.
- Collate examples of very large numbers in the real world and try to find an example for every number named.
- Perform simple calculations with very large numbers using Standard Index Form.

Extension Outcomes

Learning Points

- Be able to recognise and use factorial notation.
- Be able to calculate the number of permutations of n objects using factorials.
- Be able to calculate powers-of-powers, e.g. $2^{3^4} = 2^{81} = 2.41785\ldots \times 10^{24}$.
- Be able to understand what is meant by a mathematical limit.
- Be able to understand modular arithmetic.

Suggested Activities

- Use power towers (powers of powers of powers...) to make the largest number possible from a given set of integers.
- Experiment with the factorial button on a calculator.
- Calculate the number of different ways students in the school can arrange themselves.
- Perform simple calculations with modular arithmetic.
- Find the limits of simple convergent sequences.

Related Films

To use before the lesson plan:

Numbers: Life Without Numbers

This film explores the possibility of day-to-day living without the use of numbers.

Numbers: The Discovery of Zero

This film explains why even the simplest numbers required a leap of imagination for early civilisations.

What Does the Internet Weigh?

This film shows how numbers in standard form can be used to find a remarkably modest estimate of the weight of the internet.

To use after the lesson plan:

The Richter Scale

This film explains the link between logarithms and powers and shows how logs are used to count and measure earthquakes.

Sets: Infinity

This film introduces the concept of mathematical infinity by using Set Theory.

Cartesian Coordinates

This film provides an introduction to multidimensional cubes, the area of mathematics relevant to Graham's Number.

Can Monkeys Write Shakespeare?

This film explores the boundaries between the impossible and the improbable by calculating the probability of a monkey randomly typewriting the works of Shakespeare.

Chaos By Mistake

This film tells us why the biggest and fastest supercomputers still have problems predicting the weather.

Guide Lesson Plan

Introduction

Ask students the question at the beginning of the film: What is the largest number they can name? What is a thousand trillion called? For the largest number they come up with get them to give an example of that number in order to gain some sense of its size (e.g. there are 10^{12} fish in all the oceans, 10^{14} neural connections in the human brain, 6×10^{21} cups of water on earth...)

Show Film

The Biggest Number Ever

Main Activity

Foundation

Go through the definitions and names of powers of ten from three to ninety-nine, going up in thousands (thousand, million, billion, trillion, quadrillion, quintillion, sextillion... duotrigintillion). Get students to recognise patterns in the naming of the numbers. Explain that the googol actually has limited mathematical significance beyond being a round number. Give further examples of large numbers, e.g. letters in a DNA sequence, all of the ants in the world, all the grains of rice ever produced... Get students to try to find an example for every number defined above by multiplying quantities, e.g. hairs on a human head x number of humans, fleas on a dog x number of dogs...

Advanced

Give examples of how power towers work and then get students to calculate the biggest number they can using only powers and a given set of integers, e.g.

$$5^{6^7} = 5^{279936} > 7^{6^5} = 7^{7776}$$

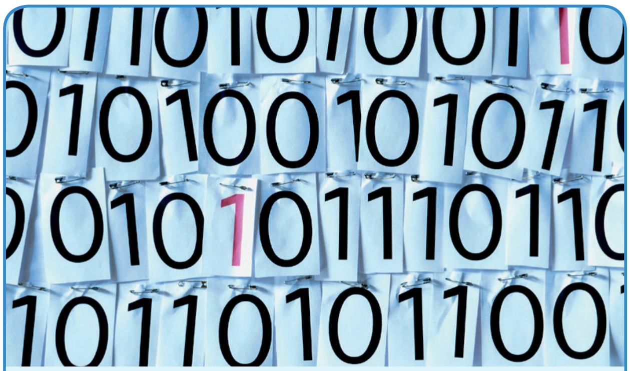
Give the very first stage of the calculation of Graham's Number and show that even this produces a number too big for a standard scientific calculator to handle.

Extension Activity

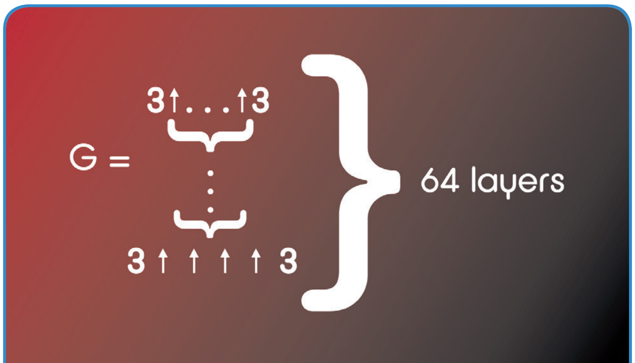
Explain that very large numbers are more useful in maths for counting possible rather than actual events. Explain factorial notation. Then suppose that the head teacher was worried about disruptive behaviour and asked teachers to look at different seating plans for 30 students in a 30-desk room. How many different arrangements would the teacher have to look at? ($30! = 2.65 \times 10^{32}$). Then look at 100 students in the sports hall, then 500 students in the main hall. If different seating plans were calculated by a computer at one per second, how long would it take to run through all permutations? Reflect on the importance of such calculations before running computer programmes.

Optional Extra

Research Knuth up-arrow notation, as shown in the film. What is $3\uparrow 3$ equal to? What is $5\uparrow 4$? Then what is $3\uparrow\uparrow 2$ equal to? And $2\uparrow\uparrow\uparrow 3$? Find other examples in mathematics where the notation is used.



Rather than writing out 100 zeros, standard form indicates the number of zeros as a power.



The world's biggest number isn't written using powers, but needs a special form of complicated notation.