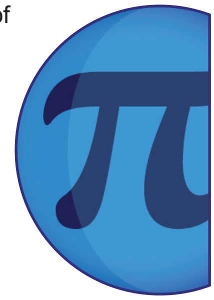
Imaginary Numbers

Key Learning Content



This film introduces complex numbers. It starts by observing that numbers in mathematics obey rules of addition, subtraction and multiplication. But the rule that a negative times a negative is positive means that the square root of a negative number does not exist as a real number. Hence mathematicians invented the imaginary number i, which is the square root of negative one. Complex numbers are then shown plotted on a graph. The use of complex numbers in solving any polynomial equation is mentioned, as is their use in modelling real world situations.

Core Outcomes

Learning Points

- Be able to understand and calculate squares and square roots.
- Be able to understand that symbols may be used to represent numbers in equations or variables in expressions and formulae.
- Be able to understand and use complex numbers.

Suggested Activities

- Draw a Venn diagram showing the relationship between natural numbers, integers, rational numbers, real numbers and complex numbers.
- Draw a graph of $y = x^2$ and use it to show why imaginary numbers were invented.
- Multiply complex numbers to form real and complex numbers.

Extension Outcomes

Learning Points

- Be able to understand that complex numbers, like algebraic expressions, follow the generalised rules of arithmetic.
- Be able to solve quadratic equations by using the quadratic formula.
- Be able to understand and use the Argand diagram.

Suggested Activities

- Add, subtract, multiply and divide complex numbers.
- Find complex solutions to quadratic equations and relate this to graphs of quadratic functions.
- Plot complex numbers $x + iy$ in the four quadrants of a graph.

Imaginary numbers make real time modelling of complex, fast-changing situations possible.

Related Films

To use before the lesson plan:

Irrational Numbers: Pythagoras	This film tells how the discovery of irrational numbers challenged the assumptions of ancient Greek mathematicians.

To use after the lesson plan:

Vectors: Air Traffic Control	This film describes the use of vectors in a real-world application of complex numbers.
Numbers: The Discovery of Zero	This film explains why the concept of the number zero was once as strange as that of complex numbers.
The Egyptians and Multiplication	This film shows how different ancient Egyptian mathematics was to the workings we use today.

Guide Lesson Plan

Introduction

Ask students to give examples of any numbers. Then ask if they have problems with any of the examples given: is anyone uncomfortable with the number zero, or with negative numbers, or with square roots of 2 or 3 or 5...? Explain that all these numbers were controversial when first introduced, raising the question: Are there any types of numbers that we don't routinely use today that will strike future generations as strange?

Show Film

Imaginary Numbers

Main Activity

Foundation

Get students to plot a graph of $y = x^2$, with x-axis from -5 to +5 and y-axis from -25 to 25, drawing a smooth curve through their plotted points. Then get them to work out how to calculate square roots of numbers from the graph, e.g. root 12, root 20. Then ask: what is root -12, or root -20? Define i as the square root of -1, then get students to calculate (root 12 x i) all squared. Using this approach, get students to calculate the roots of -13, -17, and so on.

Main Activity cont ...

Advanced

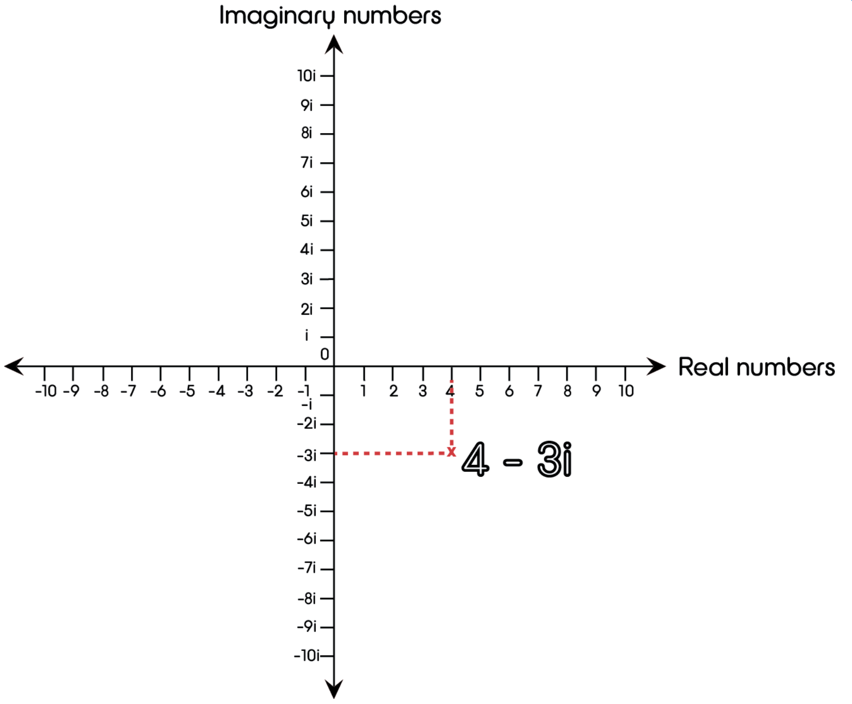
Get students to draw a graph of $y = x^2 - 5x + 6$ and then solve the equation $0 = x^2 - 5x + 6$ from the graph. Check the result using the quadratic formula. Then do the same with $y = 2x^2 + 3x + 2$ and ask students how they know from the graph that the quadratic formula will not give real roots to the equation $0 = 2x^2 + 3x + 2$.
Find complex roots using the quadratic formula.

Extension Activity

Show how to do addition, subtraction and multiplication with complex numbers and set practice exercises. Demonstrate how two complex numbers can multiply to give a real number. For advanced students, show how to divide complex numbers using complex conjugates. Multiply two complex numbers together then plot the numbers and their product on an Argand diagram and ask students if they can find any pattern with the points plotted.

Optional Extra

Ask students how many solutions they can find to the equations $x^2 = 1$ and $x^4 = 1$, both real and complex, and plot them on an Argand diagram. Then ask students how many solutions they would expect for $x^3 = 1$ and see if they can find them from the Argand diagram (hint: use symmetry).



Imaginary numbers give mathematicians values to work with which free them from the constraints that govern real numbers.