Proofs: Million-Dollar Maths

Key Learning Content

This film describes some of the most difficult problems in mathematics; so difficult that The Clay Mathematics Institute has offered prizes of $1 million for the first person to solve them. It begins by explaining that our systems of mathematics have evolved over hundreds, even thousands of years, with modern mathematicians often working on problems that are centuries old. It then lists the seven Millennium Prize problems, with brief references to the types of topic they address. The film ends with the first prize winner, a Russian mathematician who, in 2010, solved one of the seven problems, the Poincaré Conjecture. He went on to gift his prize to young students.

Core Outcomes

Learning Points

- Be able to understand the nature of mathematical proof and its link to logical reasoning.
- Be able to recognise the very broad applicability of mathematics to different aspects of the natural world.

Suggested Activities

- Work through simple mathematical proofs.
- Map different parts of mathematics to their applications in the natural world.

Extension Outcomes

Learning Points

- Be able to understand that there are different types of mathematical proof.
- Be able to prove simple mathematical results by deduction, induction and contradiction.

Suggested Activities

- Work through examples of deductive and inductive proofs, and proof by contradiction.
- Use simple propositional logic to analyse arguments.

The Millennium Prize offers $1 million for solutions to seven of the world's most difficult maths problems.

Related Films

To use before the lesson plan:

How Origami Changed the World

This film explains how a simple problem posed by the Greeks was solved thousands of years later using the Japanese art of paper folding.

To use after the lesson plan:

Proving Pythagoras

This film features one of the first and best-known mathematical theorems, and the numerous ways in which it can be proved.

Geometry: Euclid

This film shows how the idea of a formal mathematical proof was first championed by the Greek mathematician, Euclid, using examples in geometry.

Topology

This film gives an introduction to the field of mathematics that the first Millennium Prize-winner worked on when proving the Poincaré Conjecture.

A Pattern in the Primes

This film describes Riemann's Hypothesis, one of the Clay Mathematics Institute's Millennium Prize problems.

Diophantine Equations: Fermat

This film relates the story of Fermat's Last Theorem, one of the most famous theorems in mathematics, and the mathematician who proved it hundreds of years after it was first posed.

Guide Lesson Plan

Introduction

Show students the equation $y=x^2-20x+100$ and get them to work out the value of y for different values of x, which they can choose. Ask if anyone has found a negative value of y. Say you will give a prize to anyone who can find a value of x which results in a negative value of y. Ask if anyone can prove that the equation never gives a negative answer (for real values of x).

Show Film

Proofs: Million-Dollar Maths

Main Activity

Foundation

Explain that the Millennium Prizes have been offered for very complicated pieces of mathematics that are very difficult for the non-expert to even understand. Nevertheless, the process of proving a result is something that happens all the time in mathematics. Show how to prove that the equation which started the lesson is never negative, by writing it as a bracket squared. Give students other algebraic identities to prove.

Advanced

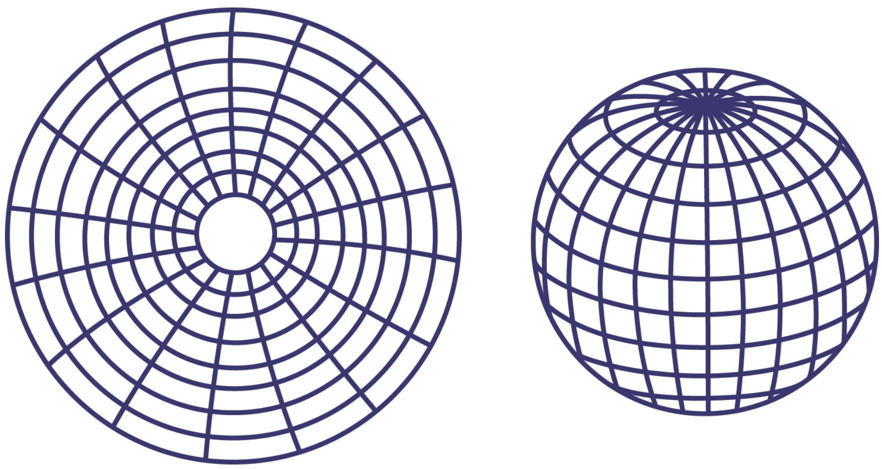
Explain what is meant by a deductive proof, proof by induction and proof by contradiction. Give examples of each type of proof, e.g. prove that the angle in a semicircle is 90 degrees (deduction); that the sum of the first n integers is given by ½ n(n+1) (deduction or induction); that root 2 is irrational (by contradiction). Set simple exercises to prove results by different methods.

Extension Activity

Give students access to the internet and get them to research the seven Millennium Prize problems with a view to identifying areas of the real world where the problems are relevant. Can they find practical problems from the real world that would be solved/would be closer to solution if the millennium problems were solved?

Optional Extra

Deductive reasoning is so important in mathematics that its rules have been formalised in propositional logic. Get students to research propositional logic and describe its key features using examples.



The Russian mathematician, Grigori Perelman, proved that there was a vital difference between a doughnut shape and a sphere.