

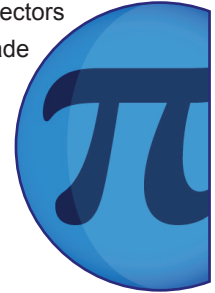


# Vectors: Air Traffic Control

## Key Learning Content

This film introduces 3D coordinates and vectors by describing the flight paths of aeroplanes. Position vectors are defined and standard notation shown on screen. The distinction between vectors and scalars is made using the example of velocity and speed. An example of vector subtraction is shown using column vectors. Parallel vectors are defined in terms of one vector being a multiple of the other.

Knowledge of 2D coordinate geometry is necessary prior to watching the film.



### Core Outcomes

#### Learning Points

- Be able to understand and use conventions for rectangular Cartesian Coordinates in three dimensions.
- Be able to locate points with given coordinates in three dimensions.

#### Suggested Activities

- Describe 3D shapes in terms of  $(x,y,z)$  coordinates.
- Calculate the distance between points in 3D space.

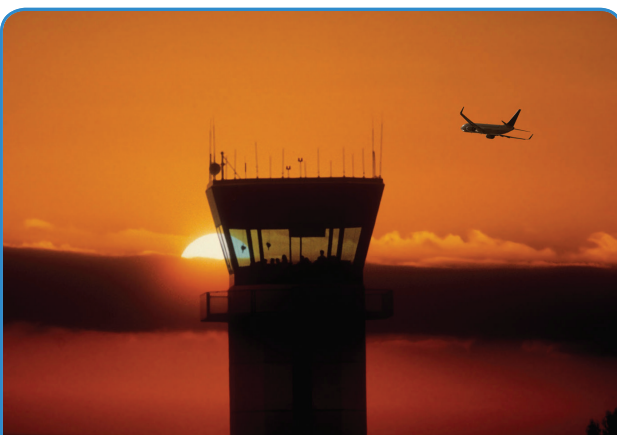
### Extension Outcomes

#### Learning Points

- Be able to understand that a vector has both magnitude and direction.
- Be able to understand and use vector notation, including column vectors.
- Be able to add and subtract vectors.
- Be able to multiply vectors by scalar quantities.

#### Suggested Activities

- Solve problems about the movement of aeroplanes using vectors.
- Describe an aeroplane flight from London to Rome using 3D vectors.



Air traffic controllers use vectors to track planes.

## Related Films

To use before the lesson plan:

### Perspective: Dazzle Camouflage

This film shows how 3D trickery helped to protect ships from submarine attacks during the war.

### Coordinate Geometry: Descartes

This film tells the story of Descartes, philosopher, mathematician and the inventor of coordinates.

To use after the lesson plan:

### Cartesian Coordinates

This film documents the ease with which 2D images can be translated to 3D, but looks at what happens when moving to 4D.

### Aiming for the Outer Planets

This film explains how astronomers arranged the convergence of the Voyager 2 space probe with different planets in order to explore deep space.

### Straight Lines: Bee Lines

This film looks at how bees seem to find the shortest distance between two points in 3D space.

## Guide Lesson Plan

### Introduction

Give a long piece of string to two students and get them to hold it taut from one corner of the room to the opposite corner; get two more students to do the same from the other corners of the room. Adjust heights so that the two strings do not touch. Imagine that the strings represent the flight paths of two planes. Ask students how they would calculate the shortest distance between the flight paths of the two planes. Explain that this is a problem that air traffic controllers have to deal with all the time.

### Show Film

### Vectors: Air Traffic Control

### Main Activity

#### Foundation

Show how  $x$ ,  $y$ ,  $z$  axes are arranged in 3D space. Give students the coordinates of a rectangular box without telling them what the shape is, and get them to work out what the shape is. Then tell them to describe a pyramid, with one base vertex at the origin, by working out the coordinates of the other vertices. Repeat for more complex shapes.

## Main Activity cont...

### Advanced

Give the starting positions of two aeroplanes together with their velocity vectors and get students to work out the positions of the planes at different times. By writing equations in terms of time, get students to work out when (if) one plane will be due north/due east/at the same height as the other (hint: consider when  $x$  or  $y$  or  $z$  coordinates are the same).

## Extension Activity

Show that Pythagoras' Theorem extends from two dimensions to three, then use it to work out the shortest distance between points in 3D space. Then set problems in terms of how long it takes planes to fly from one point to another by the shortest possible route, at different speeds.

## Optional Extra

Using an atlas which shows the height of land above sea level, map out the flight plan from one city to another across mountain ranges using 3D vectors. Work out the total distance flown between the two cities.

$$\begin{array}{c}
 B \begin{pmatrix} 2 \\ 10 \\ 4 \end{pmatrix} \\
 -A \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} = \vec{AB} \begin{pmatrix} -3 \\ 10 \\ -1 \end{pmatrix}
 \end{array}$$

To calculate the vector that describes the movement of an object such as an aeroplane between two points, the coordinates at point A are subtracted from those at point B.