Tank Wars

Key Learning Content

This film describes the use of a mathematical model to estimate the number of tanks arrayed on a battlefield. First, the serial numbers on captured tanks were examined. Then the formula $N=(M-1)(S+1)/S$ was used to estimate the total number, N, of tanks produced. M represented the largest serial number found, and S the sample size. Using this formula, the number of tanks was predicted far more accurately than by using other military intelligence methods.

While the probability theory behind the formula is beyond most school-level students, the algebraic manipulation it involves will be relevant to all.

Core Outcomes

Learning Points

- Be able to use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols.
- Be able to use correct notational conventions for algebraic expressions and formulae.
- Be able to substitute positive integers for words and letters in expressions and formulae.
- Be able to use brackets and the hierarchy of operations to multiply a single term over a bracket, and expand the product of two simple linear expressions.
- Be able to manipulate algebraic fractions where the numerator and/or the denominator can be numeric or linear.

Suggested Activities

- Use the formula to work out N for different values of M and S.
- Rewrite the formula in equivalent forms using the rules of algebra and arithmetic.
- Apply the formula to a simple game of guessing the number of bits of paper in a bag where each piece is sequentially numbered.

Extension Outcomes

Learning Points

- Be able to understand and use estimates or measures of probability from theoretical models.
- Be able to adapt formulae to take account of changes in modelling assumptions.

Suggested Activities

- Justify the formula from first principles.
- Build a spreadsheet model with random numbers to test the formula.
- Work out what changes would have to be made in the model if its underlying assumptions were changed.

Allied mathematicians guessed that tanks were being numbered chronologically, starting from 1 to 'n', the total number manufactured.

Related Films

To use before the lesson plan:

Variables: Dating By Numbers	This film describes how algebraic modelling can be used to develop a formula that predicts a person's chances of success in dating.

To use after the lesson plan:

The Birthday Paradox	This film explains how to model the probability that two people in a room share the same birthday.
Perspective: Dazzle Camouflage	This film relates how dazzle camouflage was used during both World Wars to confuse the enemy about the speed and direction of a ship, and hence make it more difficult to hit.
Beating the U-Boats	This film looks at how the properties of circles were used to help protect shipping from submarine attack during wartime.
Enigma: Cracking the Code	This film shows how German military secrets were intercepted and decoded during the Second World War.

Guide Lesson Plan

Introduction

Tell the students a story about a person visiting a friend's house and seeing some of the friend's children.
The first child said they were the third youngest in the family, the next the seventh youngest, the next the fourth youngest. How many children would you guess were in the family? Talk through the rationale for different answers.

Show Film

Tank Wars

Main Activity

Foundation

Recap the formula, then test its use by giving students small samples from different populations and getting them to work out the value of N. Then get students to multiply out the bracket and write the formula as the sum of two separate fractions. Practise algebra skills by changing the subject of the formula.

Main Activity cont ...

Advanced

Ask students to examine the formula and try to come up with an intuitively plausible explanation of why it has this form. Draw a number line on the board with numbers 1 to N and ask students what numbers you would pick so that the gap between picked numbers is always the same? Explain that $(M-1)/S$ is the 'average gap' between numbers, then ask why the result is then multiplied by $(S+1)$. Test students' understanding of the formula by asking them how it would change if you knew the numbers did not start at 1.

Extension Activity

Foundation

Get students to number small pieces of paper sequentially from 1 up, then place the pieces in a bag without anyone seeing. Get another student to draw out a sample from the bag and use the formula to predict the total number of pieces in the bag. Keep a record of actual *v.* predicted numbers.

Advanced

Use a spreadsheet to model the situation and test the formula. Use the RAND() function (or equivalent) to generate the largest number in a population by multiplying it by 100 and taking the integer value of the answer (so a random number 0.617 becomes 61.7 then 62). Then create a 'random sample' using the RAND() function again, several times, multiplying it by the maximum value calculated above and taking the integer value. Then use the formula on this sample and record how well it predicts the actual maximum. Repeat and graph the results.

Optional Extra

Get students to explain why the formula would not be appropriate to estimate the largest bus number if a sample were taken of bus numbers in a large city; nor for telephone numbers in a telephone directory. What other real-world situations would the formula be useful for?

$$N = (M - 1)(S + 1) / S$$

Imagine there are five tanks with known serial numbers. S = 5, as there are five tanks and M = 92, as that is the highest number. This gives a total prediction of 109 tanks in the army.