The Seven Bridges of Königsberg

Key Learning Content

This film tells the story of a famous problem solved by the mathematician Leonard Euler in 1735: was it possible to walk around the city of Königsberg crossing its seven bridges only once? Euler proved the challenge was impossible by developing what is now known as graph theory. By representing the problem in terms of vertices joined by arcs, Euler could define the characteristics of possible and impossible paths. The solution is shown in detail through graphics on screen, and numerous examples given.

Core Outcomes

Learning Points

- Be able to recognise the terms 'vertex' and 'arc' in the context of networks.
- Be able to describe paths through a network.
- Be able to give informal reasons for the solutions of geometrical problems.
- Be able to use and interpret maps and scale drawings.

Suggested Activities

- Work out by experimentation which networks can be traversed with one continuous non-repeating path and which cannot.
- Draw traversable networks.

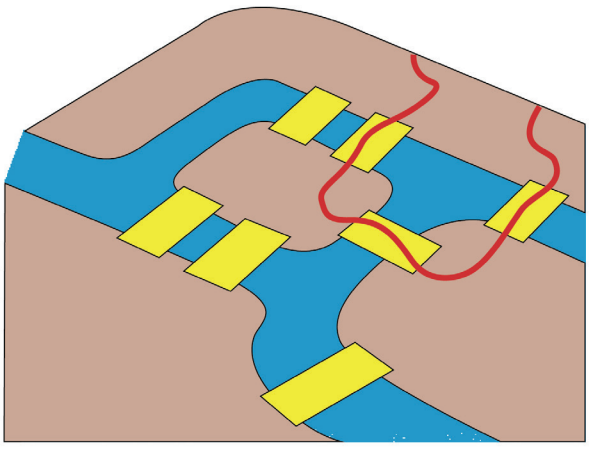
Extension Outcomes

Learning Points

- Be able to characterise networks in terms of the number and order of their vertices.
- Be able to understand that any network with none or only two odd vertices is traversable.

Suggested Activities

- Find continuous non-repeating routes through any network with exactly none or two odd vertices.
- Draw traversable networks with a given number and order of vertices.



In 1735, the Swiss mathematician Leonhard Euler proved it was impossible for people to walk around the city by crossing each bridge only once.

Related Films

To use before the lesson plan:

Networks: Labyrinths and Mazes	This film demonstrates how complex mazes can be cracked by drawing network diagrams.

To use after the lesson plan:

Degrees of Separation: Erdős	This film gives an informal introduction to social networks, and to the nature of mathematical research.
Where Is the Centre of a Triangle?	This film shows how difficult it is to answer an apparently simple question about three points and their centre.
Cartesian Coordinates	This film looks at the strange constructions that are possible when points are linked in four dimensions.

Guide Lesson Plan

Introduction

Give students a simple diagram of a square with diagonals and a triangle on top (so it looks like a simple house with a cross inside). Ask them to try to draw the diagram with one continuous path of their pen, without retracing any line. Record where successful students started and ended their paths.

Show Film

The Seven Bridges of Königsberg

Main Activity

Foundation

Give students a range of simple networks and ask them to work out which can be traversed with a continuous non-repeating path and which cannot. Get them to check that Euler's rule holds for all the networks they try. Then return to the Königsberg problem and get students to alter the town plan so that the city can be traversed by a continuous non-repeating path. Review and compare the different strategies used to achieve this.

Advanced

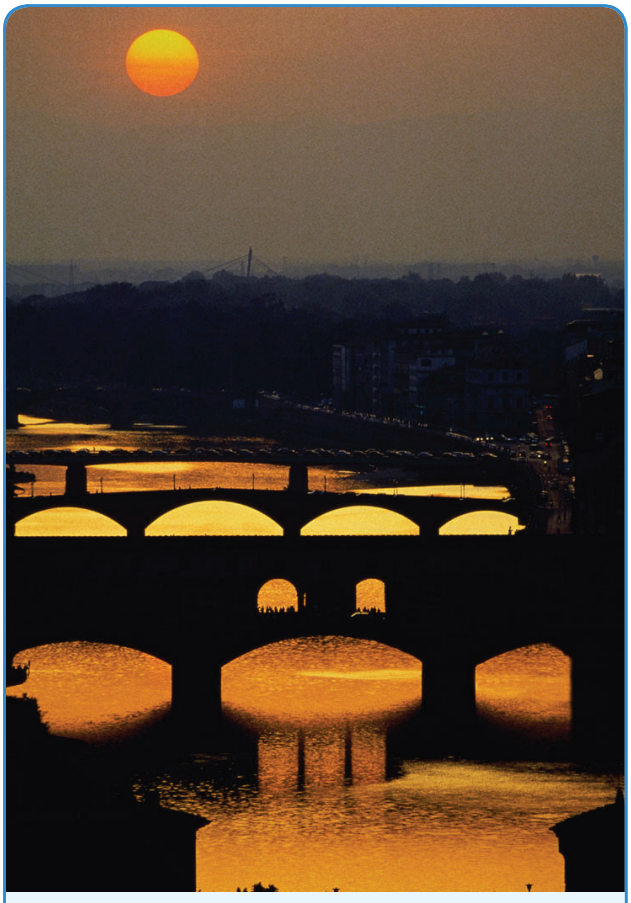
Explain that Euler's result generalises so that any network with exactly two odd vertices or none is traversable by a continuous non-repeating path. Get students to draw networks with this property and demonstrate that Euler's rule holds. Ask students whether the converse must also hold, i.e. that any network that can be traversed must then have exactly two or no odd vertices.

Extension Activity

Take maps of the local area (road maps, maps of bus routes, train routes, cycle routes) and analyse them to work out if they are traversable. Give a prize for the most complicated map of a traversable local area that students can find. Discuss how to combine maps of adjacent areas so that traversability is preserved, i.e. so that two traversable networks joined together create one larger traversable network.

Optional Extra

Euler's work set the foundations for what is now known as graph theory. Get students to research graph theory and find examples of where and how it is used today.



The problem of the seven bridges of Königsberg asked whether people could walk around the city by crossing each bridge only once.