Building the Pyramids

Key Learning Content

This film describes the challenges faced by ancient Egyptian master builders in constructing symmetrical pyramids with equal sides and angles. The 3-4-5 Pythagoras triangle is shown on screen and its special properties explained. Larger versions of this triangle are then identified in the dimensions of the pyramids. The angle of incline is defined, and then images used to demonstrate that most pyramids have the same angle of incline, as defined by a 3-4-5 triangle. Even when exceptions to this rule appear, it is still possible to explain their angles of incline using other standard Pythagorean triples.

Core Outcomes

Learning Points

- Be able to understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles.
- Be able to understand and use Pythagoras' Theorem in two dimensions.
- Be able to understand and use angles of incline or elevation, and decline or depression.

Suggested Activities

- Identify all the angles that it is possible to construct using only Pythagorean triples.
- Identify all the angles that it is possible to construct within a pyramid built in the dimensions of a 3-4-5 triangle.

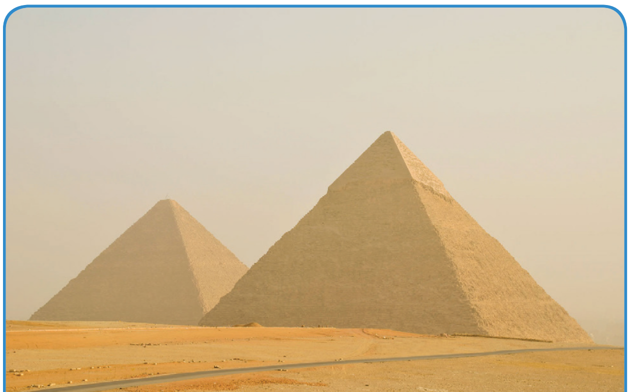
Extension Outcomes

Learning Points

- Be able to use sin, cos and tan within a right-angled triangle.
- Be able to use surds to define standard triangles used in trigonometry.

Suggested Activities

- Construct 30/60/90 and 45/45/90 right-angled triangles and work out their trigonometric ratios from first principles.
- List all the possible shapes that can be formed by taking a cross section of a pyramid.
- Find examples of standard Pythagorean triangles in other ancient buildings.



The Egyptians began the construction of every pyramid by creating a square base using 3-4-5 triangles.

Related Films

To use before the lesson plan:

The Babylonians and Plimpton 322	This film presents evidence that the Babylonians had knowledge similar to that of Pythagoras' Theorem centuries before Pythagoras was born.
Proving Pythagoras	This film introduces the famous theorem and different ways of proving it.

To use after the lesson plan:

Strengthening the Bank of China	This film shows how triangles can be central to modern architectural design.
The History of the Golden Ratio	This film looks at patterns found in various buildings, from pyramids to more recent examples.

Guide Lesson Plan

Introduction

Get students to draw from memory as accurately as they can a pyramid, as found in ancient Egypt, giving both side-on and face-on views. Then get them to measure the angle of incline of the pyramid in their drawing. Tabulate and compare results.

Show Film

Building the Pyramids

Main Activity

Foundation
Start by listing all the different Pythagorean triples that students can find with whole-number sides, which are not simply multiples of each other, e.g. 3,4,5; 5,12,13; 7,24,25… Then by drawing or by using the trigonometry buttons on a calculator, list to one decimal place all the different angles that these Pythagorean triangles allow you to construct. Shade all these angles on a number line from 0 to 90 degrees, shading two degrees either side of the angle value. Use addition and subtraction of these angles to fill the gaps in the number line as far as possible. Are there any ranges of angle which cannot be roughly approximated using Pythagorean triples?

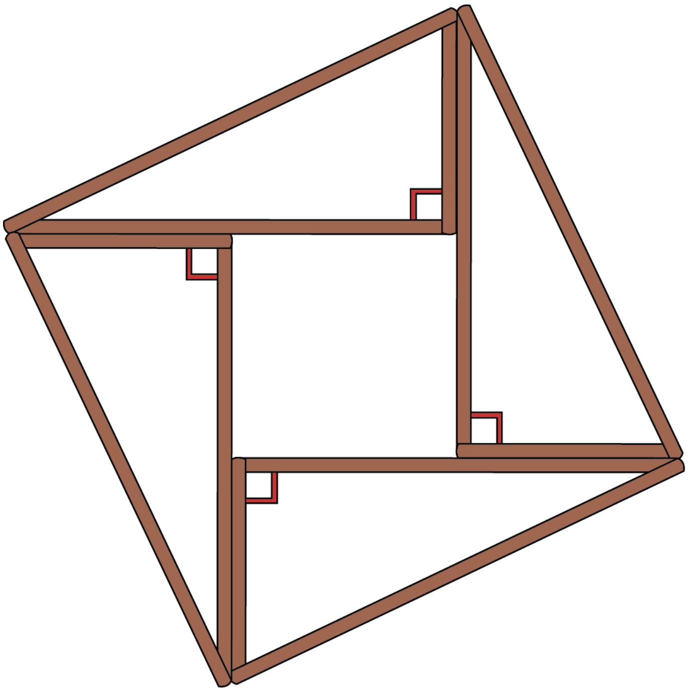
Advanced
Tell students that they are master builders and that they have been instructed to build pyramids with 30, 45 and 60 degree angles. Construct triangles with these angles using only a ruler and compass. Then, using surds, work out the exact lengths of the sides of a 45/45/90 triangle, and of a 30/60/90 triangle, given that sin(30) = ½. Explain that these two triangles are important in trigonometry and should be committed to memory.

Extension Activity

Take a pyramid based on the 3-4-5 triangle with square base ABCD and vertex V. Ask students to work out from first principles the length of AV (you may assume that V lies vertically above the centre O of the square base). Then, using trigonometry, work out the angle of incline <OAV. If a pyramid was cut in half and then slices taken through the pyramid, perpendicular to its base, describe the range of triangles that would be formed by each slice in terms of their angle of inclination.

Optional Extra

Research the construction of Stonehenge, an ancient stone circle in England, and specifically the megalithic yard that some archaeologists claim to have identified from its design. Is there evidence that Stonehenge was built using Pythagorean triples?



The base of the Great Pyramid at Khufu was created from triangles measuring 21, 28 and 35 cubits.