The Birthday Paradox

Key Learning Content

This film explains how, in a room with 23 people, it is more likely than not that two people will have the same birthday. The result is shown directly, first by considering the probability that two people have a different birthday, then three, then four, and so on. Probabilities are shown using algebra, but the final result is calculated numerically, without algebra.

Although the film leads naturally on to ideas of independent events and complementary probabilities, only a basic knowledge of probability is necessary prior to watching the film.

Core Outcomes

Learning Points

- Be able to use formulae from mathematics and other real-life contexts expressed initially in words and converting to letters and symbols.
- Be able to understand that symbols may be used to represent numbers in equations or variables in expressions and formulae, and that algebraic expressions follow the generalised rules of arithmetic.
- Be able to evaluate expressions by substituting numerical values for letters.
- Be able to manipulate algebraic fractions where the numerator and/or the denominator can be numeric or linear.

Suggested Activities

- Write the probability of same birthdays using algebra.
- Use a spreadsheet to calculate the number of people necessary in a room for the probability of a shared birthday to take a given value.

Extension Outcomes

Learning Points

- Be able to understand the language of probability in terms of 'outcomes', 'equal likelihood', 'independent' and 'complementary events'.
- Be able to determine the probability that two or more independent events will both occur.
- Be able to understand and use estimates or measures of probability from theoretical models.

Suggested Activities

- Solve problems involving independent events by multiplying probabilities.
- Solve problems involving complementary events by using p and (1-p).

The chances of two people sharing a birthday in a group could be higher than you think!

Related Films

To use before the lesson plan:

Tank Wars	This film demonstrates the use of mathematical modelling to estimate the number of tanks arrayed on a battlefield.

To use after the lesson plan:

Algorithms: Turing	This film looks at the life and work of Alan Turing, who wanted to build a computer which could think like a human being.
Can Monkeys Write Shakespeare?	This film explores the probability of a highly unlikely but still logically possible occurrence.

Guide Lesson Plan

Introduction

Check all the birthdays of the students in the room and see if any are the same; if not, get students to give the birthdays of family members until a shared birthday is found.

Show Film

The Birthday Paradox

Main Activity

Foundation

Go over the calculations shown in the film and write a formula for the probability that n people have different birthdays, using algebra. Then give students access to a computer spreadsheet program and calculate probabilities using the spreadsheet. Confirm the result (for 23 people) given in the film. Work out the number of people necessary for any given probability of a common birthday.

Advanced

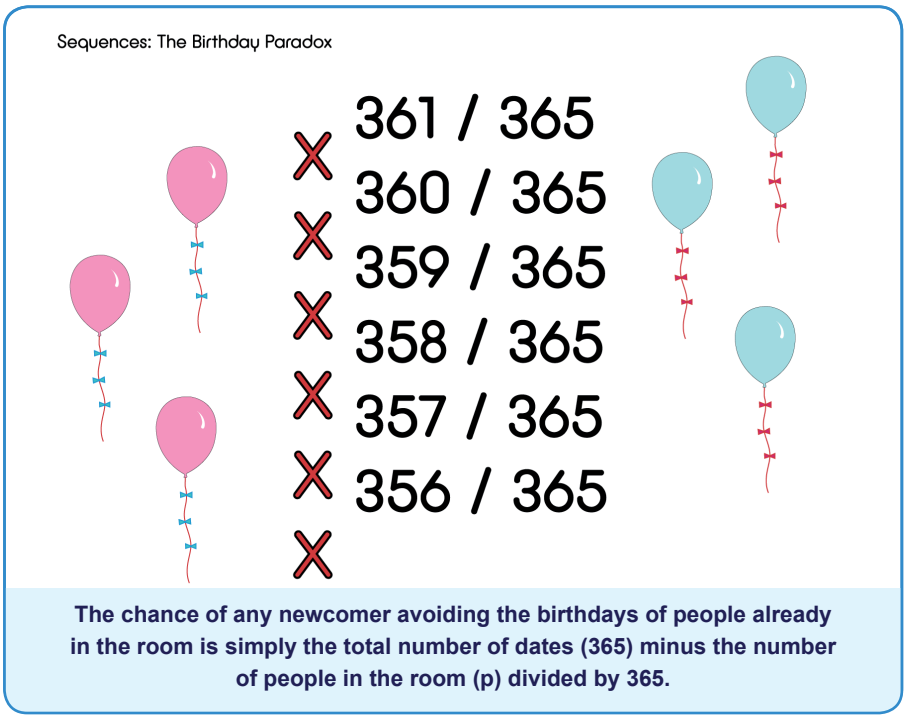
Make the distinction between dependent and independent events and test students' understanding with simple examples. Define complementary probabilities in terms of p and (1-p). Set problems involving independent events and complementary probabilities for students to solve.

Extension Activity

Ask students how many people would have to be in a room for it to be more likely than not that someone would have the same birthday as you. Explore why this is a different answer to that given in the film.

Optional Extra

Give students problems involving dependent events and ask them how they might solve them using tree diagrams.



The chance of any newcomer avoiding the birthdays of people already in the room is simply the total number of dates (365) minus the number of people in the room (p) divided by 365.